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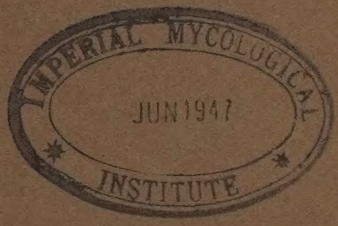

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MATHÉMATIQUES

SUR CERTAINES SINGULARITÉS QUI SURVIENNENT DANS UN SYSTÈME DYNAMIQUE QUAND L'UNICITÉ EST EN DÉFAUT

Par E. BARBACHINE

(Présenté par S. L. Soboleff, de l'Académie, le 10.VI. 1943)

Considérons un système d'équations différentielles

$$\frac{dx_i}{dt} = X_i(x_1, x_2, \ldots, x_n) \quad i = 1, 2, \ldots, n \tag{1}$$

où les fonctions $X_i(x_1, x_2, \ldots, x_n)$ sont uniformément bornées et continues par rapport à tous leurs arguments dans l'espace euclidien à n dimensions à coordonnées rectangulaires $(x_1, x_2, \ldots, x_n)$. Il passe par chaque point de R_n au moins une courbe intégrale du système (1).

En interprétant la variable t comme le temps, nous donnerons aux courbes intégrales du système (1) le nom de trajectoires. Si les seconds membres du système (1) vérifient des conditions supplémentaires qui garantissent l'unicité, alors le système (1) est un système dynamique au sens habituel. C'est M. G. Birkhoff(1) qui a étudié la structure des trajectoires d'un tel système.

Mais si on n'impose aux X_i aucune restriction sauf la continuité dans R_n, alors la construction d'une théorie analogue à la théorie générale des systèmes dynamiques de Birkhoff est encore possible, mais l'absence de l'unicité donne à cette théorie un caractère spécial.

Dans le présent article nous indiquons une voie pour la construction d'une pareille théorie et nous donnons des exemples de certaines singularités qui surviennent dans les systèmes dynamiques dans le cas où l'unicité tombe en défaut.

Nous désignerons par $\varphi(P, t)$ une trajectoire du système (1) qui sort du point P. L'arc de la trajectoire $\varphi(P, t)$, ayant pour extrémités les points de la trajectoire correspondant aux moments t' et $t'+T$, sera désigné par $\varphi(P, t', t'+T)$. La quantité T sera nommée longueur temporelle de l'arc $\varphi(P, t', t'+T)$. Le point de la trajectoire $\varphi(P, t)$ qui correspond au moment t sera désigné par $P(t)$. Nous dirons que la suite des arcs $\varphi_n(P_n, 0, T)$ converge vers l'arc $\varphi(P, 0, T)$ s'il existe un entier positif N tel que pour $n > N$ on a

$$\operatorname*{Max}_{0 \leqslant t \leqslant T} \rho(P_n(t), P(t)) < \varepsilon.$$

Il suit, en particulier, des travaux de Kamke(2) que l'on a le résultat suivant. Soit G un domaine borné et $\{P_n\}$ — une suite de points appartenant à G et convergeant vers un point P de G; supposons qu'il sort de chaque point

P_n un arc $\varphi(P_n, 0, T)$ d'une certaine trajectoire du système (1) contenu dans le domaine G. On peut choisir dans la suite des arcs $\{\varphi(P_n, 0, T)\}$ une suite partielle qui converge vers un certain arc d'une trajectoire passant par le point P.

Lemme. *Si la suite des points $\{P_i\}$ situés dans le domaine borné G converge vers un point P, alors pour chaque suite de trajectoires $\{\varphi(P_i, T)\}$ situées entièrement dans G il existe une trajectoire $\varphi(P, t)$ passant par le point P, vers laquelle converge une certaine suite partielle de la suite $\{\varphi(P, t)\}$.*

La convergence de la suite des trajectoires $\{\varphi(P_i, t)\}$ vers la trajectoire $\{\varphi(P_n, t)\}$ doit être comprise dans le sens suivant: la suite des arcs $\{\varphi(P_n, T, T)\}$ converge vers l'arc $\varphi(P, -T, T)$ pour chaque $T > 0$ fini.

Une trajectoire $\varphi(P, t)$ est nommée stable au sens de Lagrange si elle est située entièrement dans une partie bornée de R_n. Dans la suite nous ne considérerons que des trajectoires stables au sens de Lagrange.

Soit $\varphi(P, t)$ une trajectoire stable au sens de Lagrange; considérons une suite de nombres $\{t_n\}$, $\lim_{n\to\infty} t_n = \infty$. La suite $P(t_n)$ a un point limite que nous désignerons point ω-limite de la trajectoire $\varphi(P, t)$. D'une manière analogue, en considérant la suite $\{t_n\}$ quand $\lim_{n\to\infty} t_n = -\infty$ nous pouvons définir les points α-limites.

Théorème 1. *L'ensemble des points $\omega(\alpha)$-limites d'une trajectoire est fermé dans R_n et il est formé de trajectoires entières (c'est-à-dire il passe par chaque point de cet ensemble au moins une trajectoire située entièrement dans cet ensemble).*

Une trajectoire $\varphi(P', t)$ est nommée $\omega(\alpha)$-limite pour la trajectoire $\varphi(P, t)$ si pour chaque valeur finie de $T \geqslant 0$ ($T \leqslant 0$) il existe une suite d'arcs $\varphi(P, t_n, t_n + T)$, qui converge vers l'arc $\varphi(P', 0, T)$.

A la fin de cet article nous donnerons un exemple d'une trajectoire qui est formée de points ω-limites d'une autre trajectoire, mais qui n'est pas ω-limite pour cette dernière.

Un ensemble fermé compact de l'espace R_n formé de trajectoires entières est nommé minimal s'il ne contient aucun sous-ensemble fermé formé de trajectoires entières.

Théorème 2. *Chaque trajectoire stable au sens de Lagrange contient parmi ses points $\omega(\alpha)$-limites un ensemble minimal.*

Une trajectoire $\varphi(P, t)$ est nommée récurrente, si pour chaque ε il existe un nombre T tel que

$$\rho(\varphi(P, t, t+T), Q) < \varepsilon$$

pour toutes les valeurs de t et pour chaque point Q de la trajectoire $\varphi(P, t)$.

Théorème 3. *Chaque trajectoire d'un ensemble minimal est récurrente.*

Théorème 4. *Chaque trajectoire $\omega(\alpha)$-limite d'une trajectoire récurrente est elle-même récurrente.*

Remarquons que l'ensemble des points ω-limites d'une trajectoire récurrente peut ne pas être un ensemble minimal, ainsi qu'il se trouve dans la théorie des systèmes dynamiques.

Considérons dans l'espace à trois dimensions un tore dont l'équation sera écrite dans la forme paramétrique

$$x = (R + r\cos\theta)\cos\varphi, \quad y = (R + r\cos\theta)\sin\varphi, \quad z = r\sin\theta,$$

où r et R sont des constantes, et les quantités variables θ («latitude») et φ («longitude») varient entre 0 et $2\sigma_1$. Considérons sur le tore les trajectoires du système

$$\frac{d\theta}{dt} = \sigma_1 \left| \sqrt{2\sigma_1\theta - \theta^2} \right|, \quad \frac{d\varphi}{dt} = 1. \tag{2}$$

Il est facile à voir que les courbes du type $\theta = 2\sigma_1 \sin^2 \frac{\sigma_1}{2}\{t+c\}, \varphi = 2\sigma_1 \left\{\frac{t}{2\sigma_1}\right\}$, où $\{x\}$ désigne la partie fractionnaire du nombre x, sont les trajectoires du système (2), la courbe $\theta = 0$ présente aussi nne trajectoire et les points de cette courbe seront des points de non-unicité. Nous allons porter notre attention à la trajectoire

$$\theta = 2\sigma_1 \sin^2 \frac{\sigma_1}{2}\{t\}, \quad \varphi = 2\sigma_1 \left\{\frac{t}{2\sigma_1}\right\},$$

qui passe par le point $\theta = 0$, $\varphi = 0$. Montrons que cette trajectoire (que nous désignerons par Γ) est partout dense sur tore. Soit φ_0 un nombre fixe arbitraire dans l'intervalle $(0, 2\sigma_1)$. Il est évident que la trajectoire Γ doit couper la ligne $\varphi = \varphi_0$ sur le tore pour les valeurs de t vérifiant la condition

$$t = 2k\sigma_1 + \varphi_0, \quad k = 1, 2, \ldots, n.$$

L'ensemble des points d'intersection de Γ avec la ligne $\varphi = \varphi_0$ est partout dense sur cette ligne. En effet, l'ensemble des nombres $\frac{\sigma_1}{2}\{2k\sigma_1 + \varphi_0\}$ est partout dense dans l'intervalle $\left(0, \frac{\sigma_1}{2}\right)$. Mais comme la fonction $2\sigma_1 \sin^2 x$ est continue et uniforme dans $\left(0, \frac{\sigma_1}{2}\right)$, l'ensemble des nombres $\theta_k = 2\sigma_1 \sin^2 \frac{\sigma_1}{2} \cdot \{2k\sigma_1 + \varphi_0\}$, où $k = 1, 2, \ldots, n, \ldots$, est encore partout dense dans $(0, 2\sigma_1)$. Cela signifie que la trajectoire Γ est partout dense sur le tore, c'est-à-dire l'ensemble de ses points ω-limites coïncide avec tout le tore. Ainsi la trajectoire $\theta = 0$ est formée de points ω-limites, mais il est plus facile à voir qu'elle est une trajectoire ω-limite pour Γ.

On peut démontrer que la trajectoire Γ est récurrente, cependant l'ensemble des points ω-limites de Γ (c'est-à-dire le tore) n'est pas un ensemble minimal, parce que la trajectoire $\theta = 0$, qui est située sur ce tore, est fermée (dans le cas considéré c'est cette trajectoire qui présente l'ensemble minimal).

Université Lomonosoff,
Moscou.

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MATHÉMATIQUES

SUR LES OPÉRATIONS LINÉAIRES MULTIPLICATIVES

Par B. VULICH

(Présenté par A. N. Kolmogoroff, de l'Académie, le 17. VII. 1943)

Dans mes deux Notes publiées auparavant (1) j'ai construit le produit d'éléments dans des espaces semi-ordonnés linéaires contenant l'unité. Il est maintenant tout à fait naturel d'étudier les opérations que j'appelle multiplicatives. Quelques théorèmes généraux forment le sujet de cette Note, et je vais considérer dans la Note suivante la représentation analytique d'opérations linéaires multiplicatives dans certains espaces concrets. Les détails de ces travaux, de même que les résultats obtenus dans (1), seront publiés ailleurs.

1°. Dans tout ce qui suit, ainsi que dans les Notes précédentes (1), nous entendons par espace linéaire semi-ordonné un espace linéaire qui satisfait à cinq axiomes de Kantorovitch (2). D'ailleurs, on suppose toujours que l'espace contient l'unité. Soient X et Y deux espaces de ce type.

Définition. Nous appellerons une opération $y=U(x)$ transformant X dans Y multiplicative, si pour chaque paire d'éléments x_1 et $x_2 \in X$, le produit desquels $x_1 x_2$ existe, le produit $U(x_1)\cdot U(x_2)$ existe de même, et l'on a $U(x_1x_2)=U(x_1)\cdot U(x_2)$.

Nous allons indiquer quelques définitions bien connues. Une opération $y=U(x)$ appartient à la classe $H_0^0(H_0^1)$, si elle est additive et si $x_n \xrightarrow{(o)} x$ ($\xrightarrow{(t)}$) entraîne que $U(x_n) \xrightarrow{(o)} U(x)$ ($\xrightarrow{(t)}$) (on désigne par $\xrightarrow{(o)}$ la convergence ordinaire dans les espaces linéaires semi-ordonnés, et par $\xrightarrow{(t)}$ la convergence topologique). On dit qu'une opération $y=U(x)$ est positive, si l'on a $U(x)\geqslant 0$ pour tout $x\geqslant 0$.

Théorème 1. *Pour qu'une opération $y=U(x)$ de la classe H_1^1 soit multiplicative, il faut et il suffit que les conditions suivantes soient remplies:* a) *$U(x)$ est positive;* b) *si e est une quasi-unité dans X, il en est de même de $U(e)$ dans Y;* c) *si e_1, e_2 sont des quasi-unités dans X et si* inf $(e_1, e_2)=0$, *alors on a* inf $[U(e_1), U(e_2)]=0$.

Chaque opération multiplicative de la classe H_1^1 étant positive, il en résulte que, a fortiori, elle appartient en même temps à la classe H_0^0. Ceci dit, nous appellerons les opérations multiplicatives de ces classes opérations linéaires multiplicatives (op. l. m.).

Théorème 2. *Si $y=U(x)$ est une op. l. m., alors pour tout x tel que x^{-1} existe, $[U(x)]^{-1}$ existe aussi, et l'on a $U(x^{-1})=[U(x)]^{-1}$.*

Théorème 3. *Soit $\{y=U_n(x)\}$ une suite d'opérations multiplicatives. Si, pour tout $x\in X$ cette suite converge (t) et $y=U(x)$ est sa limite, alors l'opération $y=U(x)$ est de meme multiplicative.*

2°. Considérons maintenant des opérations projectives $P_e(x)$ (e est une quasi-unité arbitraire de X). J'ai démontré que $P_e(x)=xe$ [cf. la première des Notes citées sous (1), 22°]. Par conséquent, chaque opération projective est un cas particulier des op. l. m. qui transforment X en soi-même. On peut montrer à l'aide de simples exemples qu'il existe des op. l. m. qui ne sont point projectives.

Théorème 4. *Pour qu'une op. l. m. $U(x)$ qui transforme X en soi-même soit projective, il faut et il suffit qu'on ait $U(x)\leqslant x$ pour tout $x\geqslant 0$.*

Théorème 4'. *Pour qu'une opération multiplicative $U(x)$ qui transforme X en soi-même soit projective, il faut et il suffit que:* a) $U(x)$ *soit additive;* b) $U(x)$ *soit positive,* c) *on ait* $U(x) \leqslant x$ *pour tout* $x \geqslant 0$.

Nous appellerons inversible toute opération $y = U(x)$ qui prend chacune de ses valeurs une fois seulement. La condition suivante (α), qui est bien satisfaite dans certains espaces, sera utile à nous: chaque ensemble de quasi-unités contient un sous-ensemble dénombrable ou fini avec le même supremum*.

Théorème 5. *Si X satisfait à la condition (α), alors toute op. l. m. $y = U(x)$ peut être composée de deux op. l. m. à l'aide de la formule* $U(x) = U_2[U_1(x)]$, *où U_1 est une opération projective, et U_2 est inversible, son domaine étant le contredomaine de $U_1(x)$.*

On peut définir $U_1(x)$ d'une façon suivante: soit $\tilde{e} = \sup \bar{e}$, où les e sont tels que $U(e) = 0$, et soit $e = 1 - \tilde{e}$. On peut alors poser $U_1(x) = P_e(x)$. La définition de $U_2(x)$ est maintenant évidente.

3°. A. Pinsker a considéré les espaces linéaires semi-ordonnés de deux types: espaces discrets et continus. Nous allons indiquer quelques résultats concernant des opérations dans les espaces de ces types. Voici les définitions de A. Pinsker.

Une quasi-unité $e > 0$ qui n'admet pas de représentation sous la forme d'une somme de deux quasi-unités essentiellement positives $e = e_1 + e_2$ ($e_1, e_2 > 0$) s'appelle «orte». Un espace linéaire semi-ordonné contenant l'unité s'appelle continu, s'il ne contient aucun «orte». Si, au contraire, pour toute quasi-unité il existe un «orte» $e' \leqslant e$, on dit que l'espace est discret.

Tous les espaces mentionnés dans([1]) en qualité d'exemple, L^p, $\tilde{M}$, S, sont continus. Les espaces m des suites bornées de nombres réels, et s — de toutes les suites de nombres réels, sont discrets. Il est naturel de choisir en qualité d'unité dans ces espaces la suite dont tous les termes sont égaux à 1. Alors tous les «ortes» seront représentés par des suites dont un terme est égal à 1 et tous les autres à 0.

Théorème 6. *Dans chaque espace discret il existe une fonctionnelle linéaire multiplicative** qui n'est pas égale identiquement à 0. Dans les espaces continus satisfaisant à (α) une telle fonctionnelle ne peut pas exister.*

Théorème 7. *Il existe toujours une op. l. m. qui transforme un espace discret en un espace linéaire semi-ordonné quelconque contenant l'unité et qui n'est pas égal identiquement à 0. D'autre part, il n'existe aucune opération de ce type qui transformerait un espace continu satisfaisant à la condition (α) en un espace discret.*

4°. Nous allons indiquer maintenant que les op. l. m. coïncident avec une intégrale abstraite du type de Radon.

Soit X un espace linéare semi-ordonné contenant l'unité, et soit que $x \in X$. Pour chaque nombre réel λ ($-\infty < \lambda < +\infty$) désignons par e_λ l'élément caractéristique([1]) de $(x - \lambda_1)$. Il est facile de voir que e_λ varie en augmentant de 0 jusqu'à 1 quand λ varie de la même façon de $-\infty$ jusqu'à $+\infty$.

Prenons maintenant deux espaces linéaires semi-ordonnés X et Y contenant l'unité et considérons une fonction abstraite $\varphi(e)$ qui est définie pour toutes les quasi-unités $e \in X$ et dont les valeurs sont des quasi-unités dans Y. Supposons que pour chaque paire d'éléments disjoints e_1 et e_2 [c.-à.-d. que $\inf(e_1, e_2) = 0$] l'égalité suivante soit remplie: $\varphi(e_1 + e_2) = \varphi(e_1) + \varphi(e_2)$, et soit que $e_n \xrightarrow{(o)} e$ implique $\varphi(e_n) \xrightarrow{(o)} \varphi(e)$. Nous appellerons cette fonction $\varphi(e)$ fonction-mesure.

Prenons une suite de nombres réels infinie en ses deux directions: $-\infty \leftarrow \leftarrow \ldots < \lambda_{-2} < \lambda_{-1} < \lambda_0 < \lambda_1 < \lambda_2 < \ldots \rightarrow +\infty$, et supposons que $\sup_n (\lambda_n - \lambda_{n-1}) = \tilde{\lambda} < \infty$. Prenons $x \in X$ et formons la série (nous l'appellerons série de Radon) $\sigma = \sum_{-\infty}^{\infty} \lambda_n \cdot \varphi(e_{\lambda_n} - e_{\lambda_{n-1}})$.

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* Si X satisfait à la condition de régularité, (α) est, a fortiori, remplie ([2]).

** Une fonctionnelle est une opération dont les valeurs sont des nombres réels.

Nous définissons la somme de cette série, comme d'habitude, par la formule

$$\sigma = (0) - \lim_{N_1, N_2 \to +\infty} \sum_{n=-N_2}^{N_1} \lambda_n \cdot \varphi(e_{\lambda_n} - e_{\lambda_{n-1}})$$

si cette limite existe. Si la série de Radon converge pour toute décomposition de l'intervalle $(-\infty, +\infty)$, et si, pour $\tilde{\lambda} \to 0$, sa somme σ converge (0) dans Y vers une limite finie, nous dirons que l'élément x est sommable dans Y par rapport à la fonction $\varphi(e)$, et nous appellerons cette limite de σ intégrale de Radon de (x) par rapport à $\varphi(e)$ et nous la désignerons par $\int_{-\infty}^{\infty} \lambda \, d\varphi(e_\lambda)$*.

Soit $\psi(e)$ une fonction-mesure, définie pour toutes les quasi-unités dans X, dont les valeurs sont de même quasi-unités dans X. Soit $\varphi(e)$ une autre fonction-mesure avec le même domaine et dont les valeurs appartiennent à Y. Nous appellerons cette fonction $\varphi(e)$ conjuguée à $\psi(e)$, si la sommabilité de $x \in X$ dans X par rapport à $\psi(e)$ implique toujours sa sommabilité dans Y par rapport à $\varphi(e)$. Nous allons désigner par X_ψ le sous-ensemble de X composé de tous les x qui sont sommables dans X par rapport à $\psi(e)$. En conservant dans X_ψ la définition du signe $>$ donnée dans X, nous obtenons de X_ψ un espace linéaire semi-ordonné avec l'unité.

Théorème 8. *La forme générale des op. l. m. $y = U(x)$ transformant X_ψ en Y est donnée par l'intégrale de Radon $U(x) = \int_{-\infty}^{\infty} \lambda \, d\varphi(e_\lambda)$, où $\varphi(e)$ est une fonction-mesure, définie dans X, dont les valeurs appartiennent à Y, conjuguée à $\psi(e)$.*

Si l'on pose $\psi(e) = e$ dans le théorème précédent, on en obtient son résultat pour $X_\psi = X$. Si $Y = X$, l'intégrale de Radon ne représente une opération projective que quand $\varphi(e) = ee_0$, où e_0 est une quasi-unité fixe.

5°. Nous considérons dans ce paragraphe les opérations qui ne sont pas nécessairement multiplicatives. M. Kantorovitch appelle régulière une opération en espaces linéaires semi-ordonnés, si l'on peut la représenter comme la différence de deux opérations positives [cf. (2), p. 219]. Il a démontré que l'ensemble des opérations régulières est un espace linéaire semi-ordonné [(3), p. 220).

Soit maintenant $Y = X$, et supposons que cet espace contient l'unité. On voit parmi les opérations régulières transformant X en soi-même les opérations $U(x) = xy$, où $y \in X$ est un élément fixe tel que le produit xy existe pour tout x. Ces opérations forment un sous-espace linéaire semi-ordonné de l'espace $\mathfrak{A}$ de toutes les opérations régulières transformant X en soi-même**. Désignons ce sous-espace par $\mathfrak{H}$.

Théorème 9. *$\mathfrak{H}$ est un sous-espace linéaire semi-ordonné unique de l'espace $\mathfrak{A}$, qui contient toutes les opérations projectives et où l'opération identique $U(x) = x$ joue le rôle de l'unité.*

Il est à remarquer que, outre les opérations projectives, $\mathfrak{H}$ ne contient aucune op. l. m.

La Marine Baltique.

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[1] B. Vulich, C. R. Acad. Sci. URSS, XXVI, No. 9 (1940). [2] L. Kantorovitch, Rec. Math., 2 (44), 121 (1937). [3] L. Kantorovitch, ibid., 7(49), 209 (1940). [4] H. Freudenthal, Proc. Akad. Amsterdam, 39, 641 (1936). [5] S. W. P. Steen, Proc. Lond. Math. Soc., 2 (41), 361 (1936).

* L'intégrale de ce type, pour $\varphi(e) = e$, a été considérée par Freudenthal (4) et Steen (5).

** Nous supposons que dans le sous-espace sup et inf de chaque ensemble de ses elements coïncident avec ses sup et inf dans l'espace entier.

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MATHEMATICS

ON THE CONNEXION OF EXTENDED RIEMANN HYPOTHESIS WITH I. M. VINOGRADOW'S METHOD IN THE THEORY OF PRIMES

By U. LINNIK

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There are two principal methods in the theory of primes, the method of Dirichlet's L-series developed by Riemann, Hadamard, Hardy and Littlewood, and the method of Eratosphenes' sieve with ramifications towards certain combinatory inequalities (Viggo Brun, 1919) and towards multiple trigonometric sums (I. M. Vinogradow, 1937).

The second way has led, in particular, to the establishment in primes of profound properties expressed by the following inequalities (1,2):

1°. For $\alpha=\frac{Q}{q}+\frac{\theta}{q^2}$; $(Q,q)=1$; $|\theta|<1$

$$\sum_{p\leqslant N} e^{2\pi i\alpha p} \ll N^{1+\varepsilon}\left(\sqrt{\frac{1}{q}+\frac{q}{N}}+N^{-0,2}\right) \qquad (1)$$

(p denotes a typical prime) — «strong uniform distribution» of fractions (αp).

2°. Let χ be a non-principal character (mod D); k, an integer with $0<k<D$; $(k, D)=1$. Then

$$\sum_{p<N}\chi(p+k) \ll N^{1+\varepsilon}\left(\sqrt{\frac{1}{D}+\frac{D}{N}}+N^{-1/6}\right) \qquad (2)$$

— uniform distribution of values of the characters for the shifted primes.

3°. For the same χ, k and D

$$\sum_{n<N}\chi(n)\mu(n+k) \ll N^{1+\varepsilon}\left(\sqrt{\frac{1}{D}+\frac{D}{N}}+N^{-\frac{1}{6}}\right). \qquad (3)$$

It is known that the inequalities (2), (3) and (1) if proved for $k\equiv 0 \pmod D$ should be a consequence of extended Riemann hypothesis for L-series (3) (we denote it by R^* in what follows).

In the present paper a more detailed connexion of (2), (3) and (1) with R^* is established.

Namely, by using the method of generalized summation of L-series by means of Mellin's transforms (3) and with the aid of certain considerations from the theory of nearly periodic functions (4), I was able to prove the following.

Theorem. *Let $\chi_1, \chi_2, \ldots, \chi_{\varphi(D)}$ be characters for a certain D, and $L_{\chi_1}, L_{\chi_2}, \ldots, L_{\chi_{\varphi(D)}}$ the corresponding L-series. Then for any $\eta>0$ there is a corresponding constant θ, $0<\theta<\frac{1}{2}$ such that the quantity of our L-series*

possessing zeros in the rectangles $1 \geqslant \sigma \geqslant 1-\theta$, $|t| < D^{\eta/2}$, *will be* $O(D^{\eta})$.

It is easy to deduce from this theorem the inequalities of type (2); (3) and (1) with that heavy restriction, however, that instead of $D < N^{1-\varepsilon}$, $q < N^{1-\varepsilon}$, our theorem gives (2) and (3) for $D < N^{c_0}$ and (1) for $q < N^{c_0}$, where c_0 is a constant $< \frac{1}{3}$. Nevertheless, the theorem indicates a new way to (1), (2) and (3) through L-series. To explain the deduction of the inequalities of type (2) from our theorem, let $f(p)$ be an integer polynomial; X, a character (mod D); we are interested in the sum $S(N) = \sum_{p \geqslant 2} X(f(p))\, e^{-\frac{p}{N}}$. We choose an $\eta < \frac{1}{4}$ and select the number $\theta = \theta(\eta)$.

We obtain

$$S(N) = \sum_{l=0}^{D-1} X(f(l)) \frac{1}{\varphi(D)} \sum_{\alpha} \overline{\chi}(l) \sum_{p \geqslant 2} \chi(p)\, e^{-\frac{p}{N}} =$$

$$= \sum_{\chi} \frac{1}{\varphi(D)} \sum_{l=0}^{D-1} X(f(l))\, \overline{\chi}(l) \sum_{p \geqslant 2} \chi(p)\, e^{-\frac{1}{N}}.$$

Let $R = \sup_{\chi} \left| \sum_{l=0}^{D-1} X(f(l))\, \overline{\chi}(l) \right|$ for all possible χ's. A little calculation shows that for all χ's, with the possible exclusion of $O(D^{\eta})$, we shall have for $N < e^{(\ln D)^3/2}$, $\sum_{p \geqslant 2} \chi(p)\, e^{-\frac{p}{N}} \ll N^{1-\theta+\varepsilon}$. Hence we conclude that

$$S(N) \ll N \left[\varphi(D) \frac{R}{\varphi(D)} N^{-\theta+\varepsilon} + \frac{R}{\varphi(D)} D^{\eta} \right] = N \frac{R}{\varphi(D)} \left[\varphi(D)\, N^{-\theta+\varepsilon} + D^{\eta} \right]$$

The Abelian sum is, as always, more manageable than the usual one.

To explain the deduction of our theorem, we consider the most simple particular case: the evaluation of the quantity of L-series (mod D) which have zeros for $\sigma > 1 - \frac{1}{(\ln \ln D)^2}$; $|t| \leqslant 1$. It is not difficult to show that if $L(s, \chi) = \sum \chi(n)\, n^{-s}$ is such a series and $\delta = e^{-\ln D\, (\ln \ln D)^{3/2}}$, then for a set of points $s = \sigma + it$, situated in the rectangle $0 \leqslant \sigma \leqslant \frac{1}{\ln D}$, $|t| \leqslant 1$, we shall have

$$\left| \sum_{n=1}^{\infty} \chi(n)\, \Lambda(n)\, n^{-s} e^{-\delta n} \right| > A \delta^{\sigma-1} e^{-\frac{\ln D}{(\ln \ln D)^{1/4}}} > A \delta^{\sigma-1} D^{-\varepsilon}.$$

Now it is easy to calculate that

$$\sum_{\chi} \left| \sum_{n=1}^{\infty} \chi(n)\, \Lambda(n)\, n^{-\sigma+ti} e^{-\delta n} \right|^2 \ll \left(\sum \Lambda(n)\, n^{-\sigma} e^{-\delta n} \right)^2 D^{\varepsilon} \ll \delta^{2\sigma-2} D^{\varepsilon}$$

and hence the quantity of our series must be $O(D^{2\varepsilon})$.

The general case is based on analogous considerations, but requires more complicated auxiliary calculations. The detailed proof will be published hereafter.

The same method for complex characters χ (mod D) gives the formula

$$\sum_{p>2} \chi(p) \ln p\, e^{-\frac{p}{N}} \ll N \left(e^{-\frac{\ln N}{\ln D}} + \frac{1}{\ln D} \right).$$

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APPLIED MATHEMATICS

FLOW WITH SEPARATION PAST BROKEN-LINE PROFILES OF A GRATING

By N. V. LAMBIN

(Communicated by V. L. Posdunine, Member of the Academy, 20. VII. 1943)

In the plane of a complex variable $z=x+iy$ a flow of a perfect incompressible fluid, infinite, plane and stationary, is moving from the left half-plane to the right one and meets on its way a grating, that is, a totality of contours $\lambda_j (j=\pm 0, 1, 2, 3\ldots$ links of the grating) impervious to the fluid and so arranged that each one of them is obtained by a translation of another parallel to the imaginary axis for a distance multiple of a (a is the pitch of the grating).

Every link of the grating λ_j is a broken line consisting of two straight segments $P_j G_j$ and $P_j H_j$ meeting at point P_j. The points G_j are situated on the imaginary axis; the point G_0, at the origin, and the points P_j and H_j, in the right half-plane; H_j to the right of P_j. The link λ_0 lies in the fourth quadrant.

The angles α_1 and α_2 between the positive direction of the real axis and the vectors G_0P_0 and P_0H_0, respectively, are negative and less than $\frac{\pi}{2}$ in absolute value, $|\alpha_1|>|\alpha_2|$.

The flow across the grating proceeds with the separation of stream-lines at the ends G_jH_j of every link so that all of the plane z is not filled by the flow. To each link λ_j there corresponds a simply connected region M_j not filled by the flow.

This region is the one bounded by the respective link and two Jordan's curves connecting the points G_j and H_j with an infinitely remote point. For $z\to\infty$ inside M_j we have $x\to+\infty$. The angle $G_jP_jH_j$ points inward of the region M_j.

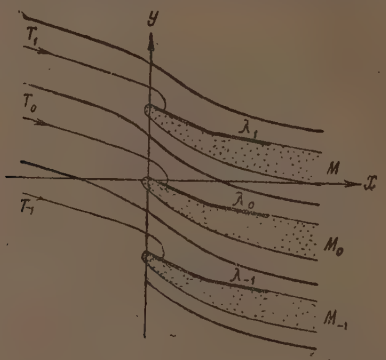
The velocity of the fluid at the boundary of the region M_j is continuous, and the separation of the stream-lines is along the tangent. As the point z moves along the contour M_j from point G_j to point H_j counter clockwise (that is, not along the link of the grating), the velocity of the flow remains constant and its direction changes monotonously. The point of separation Q_j lies between G_j and P_j (see figure).

The method of solution is this. In the plane z we separate a region bounded by two stream-lines of which one is obtained by a parallel translation of the other to a distance as large as the quantity ai. This region also includes one of the M_j regions. Its existence is ensured by the fact that the velocity of the fluid has a period ai in accordance with the shape of the grating.

Let the current filled portion T_j of this region be subjected to conformous mapping by taking for the mapping function (just as we do in solving the

analogous problem of unbroken profiles) the function $w = w(z)$, conjugated with the velocity of the fluid (Formula 1). In the plane w we then have a region bounded by the arc of a circumference having its centre at the origin, and by three rectilinear segments crossing the origin. This region is cut up along a certain line connecting a point of the circumference with an internal point of the region. What we have thus obtained in the plane w is further mapped onto a correspondingly cut region of the plane ζ, bounded by straight half-lines, parallel to the real axis, and by a segment of the imaginary axis.

This latter region we map into the first quadrant of the plane t (Formulae 2 and 3). That portion of the boundary which belongs to the grating is map-



ped thereby onto the right half of the real axis; the portion belonging to the boundary of M_j and not to the link of the grating, is mapped onto the upper half of the imaginary axis; and the remaining portion, onto the cutting line connecting the internal point t_0 of the first quadrant with the point t_1 of the imaginary axis. The point t_0 corresponds to the point $z = \infty$, $x = -\infty$, and the double point t_1, to two points $z = \infty$, $x = +\infty$ of the contour of the region T_j.

In the first quadrant of the plane t we build a hydrodynamic flow with source t_0 and absorber (negative source) t_1. This may be easily done on the principle of symmetry with respect to the coordinate axes (Formulae 4, 5 and 6). The mapping of this flow into the plane z just gives the flow sought for.

Adopting the symbols v_1 and v_2 for the velocity of the fluid at infinity before and after it meets the grating; p and q for the complex coordinates of the vertex point P_j and separation point Q_j mappings onto the plane t; and $\Phi(t)$, for the stream function of the hydrodynamical flow in the plane t, — we may write the following formulae:

$$w = w(z), \tag{1}$$

$$\zeta = \ln \frac{w \cdot e^{i\alpha_2}}{|v_2|}; \quad \ln 1 = 0, \tag{2}$$

$$\zeta = \frac{\alpha_2 - \alpha_1}{\pi} \ln \frac{t-p}{t+p} + \ln \frac{t-q}{t+q}; \quad \ln 1 = 0; \quad 0 < q < p, \tag{3}$$

$$\Phi(t) = A \ln(t - t_0) + \bar{A} \ln(t - \bar{t}_0) + A \ln(t + t_0) + \bar{A} \ln(t + \bar{t}_0) - \\ - (A + \bar{A}) \ln(t - t_1) - (A + \bar{A}) \ln(t - \bar{t}_1), \tag{4}$$

where

$$A = \frac{\bar{a} v_1}{2\pi} \tag{5}$$

and

$$\arg \frac{t_1 - t_0}{q - t_0} + \arg \frac{t_1 + t_0}{q + t_0} = -\frac{\pi}{2} - \arg A. \tag{6}$$

Eliminating ζ from (2) and (3) and joining the formula

$$\frac{dw}{dt} = \frac{\Phi'(t)}{\overline{w(t)}}, \qquad w(t) = w[z(t)], \tag{7}$$

which follows directly from the definition of w and Φ, we get a set of formulae that solve the problem.

The solutions are obtained when the parameters α_1, α_2, p, q, t_0, t_1 and a are assigned arbitrary values satisfying the physical conditions of the problem, as well as the requirement that whether $\arg v_1$ is calculated from (5) and (6) or from (2) and (3) (for $t = t_0$), the result should be invariable.

The value of w corresponding to the point z is derived from (2) and (3), while the point z itself is obtained by integrating the equation (7) from 0 to t.

The lengths of the segments $P_j G_j$ and $P_j H_j$ are likewise found by the integration of (7) from 0 to p and from p to ∞, respectively.

The need for solving this problem has arisen in connexion with the theory of supercavitating screw propellers, which is being worked out by V. L. Posdunine [1].

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HYDRODYNAMICS

STREAM-LINE MOTION OF FLUIDS PAST BODIES OF REVOLUTION

By J. M. SEREBRISKY

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The present work is concerned with a method for the calculation of the axial and transverse flow of an ideal incompressible fluid past bodies of revolution. By means of it the distribution of pressure upon the surface of a body of revolution, as well as the field of velocities around the latter may be built with great precision. The method is to be applied in calculating the flow past bodies shaped like an airship, smooth contours of fuselage bows, etc.

§ 1. Fundamental Equations. We shall consider the plane of a meridional section of a body of revolution and proceed in that plane from the usual Cartesian coordinates (x, y) to elliptical coordinates (ψ, θ) [5]. We have

$$x = \operatorname{ch}\psi \cos\theta; \quad y = \operatorname{sh}\psi \sin\theta. \tag{1.1}$$

Let us examine the case of an axial flow when the velocity upstream is parallel to the axis of rotation of the body. If the notation $\lambda = \operatorname{ch}\psi$; $\mu = \cos\theta$ is adopted, the equation for the velocity potential Φ in coordinates λ, μ takes the form:

$$\frac{\partial}{\partial\lambda}\left[(\lambda^2-1)\frac{\partial\Phi}{\partial\lambda}\right] + \frac{d}{\partial\mu}\left[(1-\mu^2)\frac{\partial\Phi}{\partial\mu}\right] = 0. \tag{1.2}$$

The general solution of this equation is known to be representable as a series [1-3] $\Phi = \sum_{n=0}^{\infty} A_n P_n(\mu) Q_n(\lambda)$, where $P_n(\mu)$ are Legendre functions of the first kind, and $Q_n(\lambda)$ are the associated functions of Legendre [4]. This series converges outside a certain ellipsoid drawn through a singular point that is located inside the body and is the most remote from the axis of rotation (having the largest coordinate λ). In particular, if the singularities are situated on the segment $-1 \leqslant x^* \leqslant +1$ of the axis, the series converges in the whole space outside this segment. Making use of the equation for the stream function [2,3], we find that the following fundamental relation must be satisfied on the surface of the body:

$$\sum_{n=1}^{\infty} \frac{2A_n}{n(n+1)} \frac{dP_n}{d\mu} \frac{dQ_n}{d\lambda} + 1 = 0. \tag{1.3}$$

If the A_n are known, the ratio of the local velocity (v) to that of the upstream flow (v_∞) can be determined for any point from the formula

$$\left(\frac{v}{v_\infty}\right)^2 = \frac{1}{\lambda^2-\mu^2}\left[(\lambda^2-1)\left(\sum_{n=1}^{\infty} A_n P_n(\mu)\frac{dQ_n}{d\lambda}+\mu\right)^2 + \right.$$

$$\left. +(1-\mu^2)\left(\sum_{n=1}^{\infty} A_n \frac{dP_n}{d\mu} Q_n(\lambda)+\lambda\right)^2\right]. \qquad (1.4)$$

§ 2. Development of Legendre Functions. For thin bodies of revolution with smooth outline the value of $\lambda = \operatorname{ch}\psi = 1+\frac{\psi^2}{2}+\ldots$ is very near to unity. Here are two auxiliary developments:

$$\frac{1}{1-\lambda^2} = -\frac{1}{\psi^2}+\frac{1}{3}-\frac{\psi^2}{15}+\ldots; \quad \frac{1}{2}\ln\frac{\lambda+1}{\lambda-1} = \ln\frac{2}{\psi}+\frac{\psi^2}{12}+\ldots \qquad (2.1)$$

For comparatively small ψ (of order $0<\psi<0.4$) the quantities $\frac{1}{\psi^2}$ and $\ln\frac{2}{\psi}$ are the main terms of the expression in (2.1), while the rest are slight corrections. By means of (2.1) we obtain the following development for Q_n and $\frac{dQ_n}{d\lambda}$:

$$\left.\begin{aligned} &Q_0 = \ln\frac{2}{\psi}+\ldots; && Q_1 = \ln\frac{2}{\psi}-1+\ldots;\\ &Q_2 = \ln\frac{2}{\psi}-\frac{3}{2}+\ldots; && Q_3 = \ln\frac{2}{\psi}-\frac{11}{6}+\ldots;\\ &\ldots\ldots\ldots\ldots\ldots\ldots && \end{aligned}\right\} \qquad (2.2)$$

$$\left.\begin{aligned} &\frac{dQ_0}{d\lambda} = -\frac{1}{\psi^2}+\frac{1}{3}+\ldots; && \frac{dQ_1}{d\lambda} = -\frac{1}{\psi^2}+\ln\frac{2}{\psi}-\frac{1}{3}+\ldots;\\ &\frac{dQ_2}{d\lambda} = -\frac{1}{\psi^2}+3\ln\frac{2}{\psi}-\frac{17}{6}+\ldots; && \frac{dQ_3}{d\lambda} = -\frac{1}{\psi^2}+6\ln\frac{2}{\psi}-\frac{47}{6}+\ldots;\\ &\ldots\ldots\ldots\ldots\ldots\ldots && \end{aligned}\right\} \qquad (2.3)$$

Making use of the recurrent formula $(n+1)Q_{n+1} = (2n+1)\lambda Q_n - nQ_{n-1}$, it may easily be shown that in all Q_n and $\frac{dQ_n}{d\lambda}$ the first terms of the expansion are independent of n. Let us assume that $Q_n = \ln\frac{2}{\psi}+\gamma_n$; $\frac{dQ_n}{d\lambda} = -\frac{1}{\psi^2}+\delta_n$. The less is ψ, the smaller γ_n and δ_n in comparison with the first terms of the expansion.

It is known from the theory of the Legendre functions [4] that $P_n(\cos\theta)$, $\frac{dP_n}{d\mu}$ and their linear combinations may always be replaced by a finite trigonometric series. In the sequel we will have to represent the different functions $F(\theta)$ in the form of the series $F(\theta) = \sum_{n=1}^{\infty} a_n \frac{dP_n}{d\mu}$. We prefer, however, to develop $F(\theta)$ in the ordinary trigonometric series $F(\theta) = \sum_{n=1}^{\infty} b_n \cos(n-1)\theta$, and then, taking for n an arbitrary finite value N; pass from b_n to a_n. Putting, for instance, $N=5$, we get the following formulae:

$$\left.\begin{aligned} &a_1 = b_1-\frac{3}{5}b_3+\frac{3}{35}b_5; && a_2 = \frac{1}{3}b_2-\frac{3}{7}b_4;\\ &a_3 = \frac{4}{15}b_3-\frac{16}{45}b_5; && a_4 = \frac{8}{35}b_4; \quad a_5 = \frac{64}{315}b_5. \end{aligned}\right\} \qquad (2.4)$$

§ 3. Axial Flow past Bodies of Revolution. In solving eq. (1.3), put $\frac{dQ_n}{d\lambda} = -\frac{1}{\psi^2} + \delta_n$ and take that the quantity δ_n may be neglected in first approximation. Then we obtain the following equation, whereby the problem of the axial flow past a thin revolution body of smooth outline is solved in elliptical coordinates:

$$\psi^2 = \sum_{n=1}^{\infty} \frac{2A_n}{n(n+1)} \frac{dP_n}{d\mu}. \tag{3.1}$$

If the notation $a_n = \frac{2A_n}{n(n+1)}$ is introduced, it remains merely to develop ψ^2 in a trigonometric series $\psi^2 = \sum_{n=1}^{\infty} b_n \cos(n-1)\theta$ and to pass from b_n to a_n.

Computations have shown that for smooth and not very thick bodies of revolution first approximation is accurate enough for practical purposes. The error is seldom higher than 0.1—0.5 per cent of the velocity. And the second approximation is simply obtained by the introduction of slight corrections in the first.

The formulae for these corrections are derived as follows. Let $a_n = a_{n1} + \Delta a_n$, where a_{n1} is the value of a_n in first approximation. Then, ignoring the product $\Delta a_n \delta_n$, we get

$$\psi^2 \sum_{n=1}^{\infty} a_{n1} \frac{dP_n}{d\mu} \delta_n = \sum_{n=1}^{\infty} \Delta a_n \frac{dP_n}{d\mu}; \tag{3.2}$$

and the matter is reduced to the determination of the function $\psi^2 \sum_{n=1}^{\infty} a_{n1} \frac{dP_n}{d\mu} \delta_n$ and its subsequent expansion in a series for determining Δa_n. Geometrically the introduction of the correction means that the calculation in first approximation is made for a somewhat different body from that given. In the case of thick bodies (with ψ about 0.2—0.5) a still better first approximation is obtained, if the function

$$M(\psi) = \left(\frac{1}{\psi^2} - \ln\frac{2}{\psi} + \frac{1}{3}\right)^{-1} = \psi^2 + \psi^4\left(\ln\frac{2}{\psi} - \frac{1}{3}\right) + \ldots$$

is developed instead of ψ^2.

If we shall be content with the first approximation, we need not overcome the difficulties that are always presented by an axially symmetrical problem in constructing a body according to some given A_n. It may be remarked that if the first approximation is adopted in the form (3.1), an increase of the relative thickness of the body by k times may be assumed to increase the coefficient A_n k^2 times, and this very accurate assumption essentially facilitates the design of a series of bodies of different thickness (5).

§ 4. Transverse Flow past Bodies of Revolution. Let us now consider the problem of a transverse movement of fluid past a body of revolution when the velocity of the upstream flow is supposed to be directed at right angles to the axis of rotation of the body. Kaplan (2) has shown that in this case the following fundamental condition should be fulfilled on the surface of the body:

$$\sum_{n=1}^{\infty} C_n \left\{ \frac{dP_n}{d\mu} \frac{dQ_n}{d\lambda} - \left[\frac{d(\lambda\mu)}{d\mu}\right]^{-1} n(n+1) \frac{d}{d\mu}(P_n Q_n) \right\} - 1 = 0. \tag{4.1}$$

It may be proved that the expression $L_n = \left[\frac{d(\lambda\mu)}{d\mu}\right]^{-1} n(n+1) \frac{d}{d\mu}(P_n Q_n)$ is

a quantity of order δ_n. By ignoring in first approximation all quantities of order δ_n, we then obtain the following equation by which the problem of a transverse flow past a thin revolution body of smooth outline is solved in elliptical coordinates:

$$\psi^2 = -\sum_{n=1}^{\infty} C_n \frac{dP_n}{d\mu}. \qquad (4.2)$$

It turns out that the coefficients A_n and C_n are connected by simple relation, for $C_n = -\frac{2A_n}{n(n+1)}$. The correction for second approximation is easily introduced by analogy with (3.2). Calculations, however, have shown that in the case of the transverse flow; too, the first approximation is sufficient for practical purposes.

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PHYSICS

COULOMB FORCES AND STRUCTURE OF THE NEUTRON

By I. POMERANCHUK

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According to the concepts of the meson theory of nuclear forces, the field of nuclear forces must be associated with charged particles, the mesons; by exchanging these, heavy elementary particles mutually act upon one another. For the present the properties of mesons are known but little. From the experimental data available [1-3] it may, however, be concluded that the spin of mesons cannot be equal to 1. It is thus necessary to consider particles with a whole spin, unless we are to adopt an exchange of meson pairs at once. If we exclude spin 1, two possibilities are left at our disposal, the so-called scalar and pseudoscalar mesons.

In the absence of sources (of protons or neutrons) the meson field with a spin 0 is determined by the Klein-Gordon-Schrödinger-Fock equation:

$$\left(\frac{\hbar c}{i}\nabla + e\vec{A}\right)^2 \psi + m^2 c^4 \psi = \left(-\frac{\hbar}{i}\frac{\partial}{\partial t} - e\varphi\right)^2 \psi, \qquad (1)$$

where $\hbar$ is the Plank constant divided by 2π; c; the velocity of light; m, the resting mass; $\vec{A}$, the vector potential of the electromagnetic field, and φ, its scalar potential.

Let us apply equation (1) to the analysis of the meson field surrounding the heavy particles. It is obvious that for the case of a proton; φ and $\vec{A}$ should be put equal to 0, the general charge of the system being equal to unity; this single charge is now attached to a heavy particle in the state of proton, and then to the meson field, while the heavy particle is in the state of a neutron. The meson field around the proton is thus determined by the equation:

$$-\hbar^2 c^2 \Delta\psi + m^2 c^4 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = \varepsilon^2 \psi. \qquad (2)$$

ε describes the dependence of ψ on time:

$$\psi = \psi_0 e^{-i\varepsilon t/\hbar}. \qquad (3)$$

It is evident that in (2) ε may assume any value. The value of ε is determined from the difference between the masses of a proton and a neutron, because the density of the sources ρ of a meson field is in the usual way composed of the wave functions of a proton Φ_p and a neutron Φ_n:

$$\rho \approx \Phi_n^* L \Phi_p = \Phi_{n_0}^* L \Phi_{p_0} e^{i\frac{(E_n - E_p)t}{\hbar}} \quad (4)$$

where L is an operator which depends on whether the mesons are scalar or pseudoscalar ([4],[5]). What is essential; this operator contains no factor dependent on time. By combining (3) and (4), we obtain:

$$|\varepsilon| = |E_n - E_p| . \quad (5)$$

It turns out that ε may be uniquely determined from (1), if (1) is applied to describe the meson field surrounding the neutron. Unlike the case of proton, φ and $\vec{A}$ are now not equal to zero, since the meson field and the heavy particle have their general charge equal to zero; the neutron is thereby transformed into a proton and a negative meson, which are in electromagnetic interaction. Let us apply (1) to this case, putting $\vec{A}$ in the first approximation equal to zero, and $\varphi = \frac{e^2}{r}$, where r is the distance to the source of the field; e, the charge of the electron. The solution of the equation thus obtained has been more than once investigated [see e. g. ([6])]. In states with the least energy ψ_0 has the shape:

$$\psi_0 = \text{const}\, r^\gamma e^{-\frac{r}{\hbar c}\sqrt{m^2c^4 - \varepsilon^2}} \quad (6)$$

$$\gamma = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \alpha^2} \quad (7)$$

(the orbital moment $l = 0$). The value of γ corresponding to the negative value of the root is usually omitted. Indeed, this solution has no bearing on the problem of the hydrogen atom. But in our case it is this solution that gives us the meson field around the neutron:

$$\gamma = -\frac{1}{2} - \sqrt{\frac{1}{4} - \alpha^2} \approx -1 + \alpha^2 . \quad (8)$$

As for ε, with such a γ it is equal to $\frac{e^2}{\hbar c} mc^2$ (with an accuracy up to α^3):

$$-\varepsilon = \frac{e^2}{\hbar} mc^2 . \quad (9)$$

This quantity expresses surprisingly well the difference between the masses of a proton and a neutron in both magnitude and sign when $m \approx 200\, m_0$, where m_0 is electron mass*. By means of (9) and (8) we find ψ_0:

$$\psi_0 \approx \text{const}\, \frac{1}{r^{1-\alpha^2}} e^{-\frac{rmc}{\hbar}\left(1 - \alpha^2 \frac{1}{2}\right)} . \quad (10)$$

This expression coincides with an accuracy up to magnitudes $\sim \alpha^2$ with the «usual» meson potential in the case of scalar mesons, which is obtained e. g. by applying the method of the perturbation theory (Yukawa). Hence it follows that it is just the γ of (8) that must be taken in order to obtain the meson field of a neutron.

Until now we have neglected the action of the magnetic field of the proton spin. Taking into account the spherical symmetry of ψ_0 and comparing to the proton Dirac's value of the magnetic moment, the following equation is obtained:

* $m \approx 200\, m_0$ is the most plausible result of experimental measurements of the meson mass.

$$-\hbar^2 c^2 \Delta\psi + m^2 c^4 \psi + \frac{e^2\hbar^2}{4M^2c^2}[\vec{s}\,\vec{r}]^2\psi = (\varepsilon - e\varphi)^2\psi. \qquad (11)$$

s_x, s_y, s_z are Pauli's matrices*. M is the mass of the proton. Considering the last term on the left-hand side as a small perturbation, we find $\Delta\varepsilon$:

$$\Delta\varepsilon = \frac{e^2\hbar^2 \int \psi^* \frac{[\vec{s}\,\vec{r}]^2}{r^3}\psi\, d\tau}{8M^2c^2 \int \psi^* \left(\varepsilon_0 + \frac{e^2}{r}\right)\psi\, d\tau}. \qquad (12)$$

With small r the numerator in (12) diverges as $\frac{1}{r^3}$. It is obvious that $r_{\min} \gg \frac{\hbar}{Mc}$, because with $r \leqslant \frac{\hbar}{Mc}$ the field of the spin will be considerably less than if found according to $\vec{A} = \frac{e\hbar}{2Mc}\frac{[\vec{s}\,\vec{r}]}{r^3}$:

$$-\Delta\varepsilon \ll \frac{2}{9}\frac{e^4\hbar^2M^2c^2}{8M^2c^2e^2\hbar^3\left(\ln\frac{M}{m}+1\right)} = \frac{\alpha Mc^2}{3\cdot 36} \approx 70 \text{ kV}.$$

The position of the neutronic level in a scalar meson is thus subject to no essential change by magnetic interaction.

We see that the difference between the masses of a neutron and a proton may be explained directly from the behaviour of scalar mesons.

If the case of pseudoscalar mesons is to be treated correspondingly, it is necessary to take states with an orbital moment $l = 1$. Instead of (9) we obtain here ε equal to

$$\varepsilon_0 < 0; \quad |\varepsilon_0| = \frac{1}{3}\alpha mc^2 \qquad (13)$$

(the magnetic field of the spin being not taken into account).

The magnetic field of the spin makes the matter still worse. By applying to (1) the theory of perturbation and allowing for the presence of orbital moment we have:

$$\Delta\varepsilon = -\frac{e^2\hbar}{2Mc}\hbar c \frac{\int \psi^* s_z \frac{1}{i}\frac{\partial\psi}{\partial\varphi}\frac{d\tau}{r^3}}{\int \psi^*\left(\varepsilon_0 - \frac{e^2}{r}\right)\psi d\tau} \qquad (14)$$

The expression for $\Delta\varepsilon$ diverges for small r. For the same reason as in (12); $r_{\min}$ should be $\gg \frac{\hbar}{Mc}$. Taking into account that with $r \ll \frac{\hbar}{mc}$ ψ has the shape const $\frac{e^{-\frac{rmc}{\hbar}}}{r^2} p(\cos\vartheta, \varphi)$; the following expression can easily be obtained:

$$\Delta\varepsilon = e^2\frac{\hbar}{2Mc}\frac{\hbar c}{e^2} s_z \mu_z \frac{2}{4r^2_{\min}};$$

where μ_z is the magnetic quantic number of the meson. If, as it is usually assumed, the spin of a neutron is equal to $\frac{1}{2}$, then with $s_z = 1$, $\mu_z = 1$, since in the case of a negative charge the proper values are multiplied by the negative density. With $\mu_z = 1$ the projection of the moment of a meson will be $-\hbar$, which together with $s_z = 1$ gives a full moment of the neutron equal to $\frac{1}{2}\hbar$.

$$\Delta\varepsilon = e^2 \frac{\hbar}{8Mc}\frac{1}{\alpha}\frac{1}{r^2_{\min}}$$

* They act upon the spin variable of the proton.

$r_{\min}$ is the minimum distance up to which the integration in (14) has a sense: $\frac{\hbar}{Mc} \ll r_{\min} < \frac{\hbar}{mc}$. But even if $r_{\min}$ is substituted for $\frac{\hbar}{mc}$, we obtain for $\Delta\varepsilon$:

$$\Delta\varepsilon > 2e^2 \frac{\hbar}{8Mc} \frac{1}{\alpha} \frac{m^2c^2}{\hbar^2} = \frac{m}{4M} mc^2 = \frac{mc^2}{36} = 2.8\,\text{MV} \quad [-\varepsilon_0 \approx 250\ \text{kV}].$$

Essential is that $\Delta\varepsilon > 0$, therefore $\Delta\varepsilon + \varepsilon_0 > 0$, i. e. a neutron must have a mass smaller than a proton. Pseudoscalar mesons give an incorrect value for the difference between the masses of a neutron and a proton.

Turning now to the anomalous moments of a neutron and a proton, it should be noted that the moment given by pseudoscalar mesons is nearly 100 times greater than the required one. The anomalous moment of k proves equal to

$$k = + \frac{e\hbar c}{2} \frac{\int \psi^* \frac{1}{\tau} \frac{\partial \psi}{\partial \tau} d\tau}{\int \psi^* \left(-\frac{e^2}{r} + \varepsilon_0 \right) \psi d\tau} = \frac{e\hbar c}{e^2} r_{\min} \mu_z .$$

Even if $r_{\min} = \frac{\hbar}{Mc}$, $|k| = 274\, k_0$ (k_0 is the proton's magnetic moment according to Dirac). As a matter of fact; $r_{\min}$ is probably greater than $\frac{\hbar}{Mc}$; at any rate, if $r_{\min} < \frac{\hbar}{Mc}$, the motion of the proton should be taken into account. Along with an incorrect value of the neutron's mass, pseudoscalar mesons give an incorrect order of magnitude for the anomalous magnetic moment.

It should be pointed out that if the spin of the neutron was equal to $\frac{3}{2}$, the sign of mass difference in all pseudoscalar mesons would be correct [$\Delta\varepsilon$ in (14) would be < 0].

We arrive at the conclusion that the pseudoscalar mesons alone cannot account for the properties of the neutron. Turning back again to scalar mesons, it should be said that, contrary to what is usually believed, they give an anomalous magnetic moment of a correct order of magnitude, though evidently different from the neutronic one. The existence of such a singular moment is due to the fact that the meson state is acted upon by the magnetic field of the proton spin. Indeed, in the case of an external magnetic field we instead of (11) have:

$$-\hbar^2 c^2 \Delta\psi + m^2 c^4 \psi + \frac{e^4\hbar^2}{4M^2c^2r^6} [\vec{s}\,\vec{r}]^2 \psi + \frac{e^3\hbar}{2Mc} ([\vec{s}\,\vec{r}][\vec{B}\,\vec{r}]) \frac{\psi}{r^3} = (\varepsilon - e\varphi)^2 \psi$$

(the spherical symmetry of ψ should be kept in mind). The last term on the left-hand side gives a magnetic moment equal to:

$$Bk = -\frac{e^3\hbar \int \psi^* ([\vec{s}\,\vec{r}]\,[\vec{B}\,\vec{r}]) \psi \frac{d\tau}{r^3}}{4Mc \int \psi^* \left((\varepsilon_0 - \frac{e^2}{r}) d\psi \tau \right)} = \frac{1}{6} e \frac{\hbar}{Mc} \frac{B}{\ln \frac{M}{m} + 1} .$$

The moment obtained is in disagreement with experimental data, although it is correct as to the order of magnitude. We are confronted here with an essential difficulty. Perhaps, it is expedient to consider the interaction between the heavy particles and several mesons at once.

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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TECHNICAL PHYSICS

A NEW STANDARD OF WAVE-LENGTH

By L. B. PONIZOVSKY

(Communicated by V. P. Linnik, Member of the Academy, 21. VI. 1943)

The cycle of investigations undertaken in the State Optical Institute with the object of finding a new spectral standard of length was interrupted by the war. But as this problem was discussed recently at the American Conference on Spectroscopy, Chicago 1942 (1), it may now be of interest to publish the results we had obtained toward the beginning of 1941.

It was at the suggestion of A. A. Lebedev that we had entered upon the search for a primary standard of wave-length in the infra-red region of the spectrum.

What may be said in favour of the infra-red spectral lines in relation to the length measurement by the interference method is this:

1) Going down into the infra-red region will give a sharper pattern of interference because of a certain decrease of absorption and increase of the reflection coefficient of the metal films coating the plates of the etalon.

2) Increase of the least reference etalon ($t=\frac{N\lambda}{2}$), with the order of interference N unchanged, will simplify the measurement of large lengths by the method of comparing the multiple etalons to the initial etalon measured directly in wave-lengths.

3) The distance between two neighbouring rings (interference fringes) in the focal plane of a lens is proportional to the wave-length

$$\Delta R=\frac{\lambda f^2}{2Rt},$$

where f is the focal distance of the lens; t, distance between the mirrors in a Fabry-Perot etalon; R, radius of the ring.

In other words, the rings of shorter wave-length are closer together than those of longer wave-length.

4) The distance in wave-lengths between two neighbouring maxima is proportional to the square of the wave-length:

$$<\Delta\lambda>=\frac{\lambda^2}{2t}.$$

This will mean that a larger λ allows the use of a broader spectral line without decreasing the sharpness of the interference pattern.

Such are the advantages with the infra-red lines used as length standards. But there are also some drawbacks. Thus; it should be noted that the high therms

of the atom are more influenced by electric and magnetic fields. The Doppler broadening grows with the wave-length. And, finally, working with an invisible line makes focusing, photographing and the interpretation of the interference pattern more complicate.

To overcome these difficulties has been the aim of the present work.

For studying the infra-red lines suitable for the interference measurement of length a set-up was mounted according to the scheme represented in Fig. 1. The source of light S was a hollow cathode tube, designed by Usikov([2]). Feeding of the

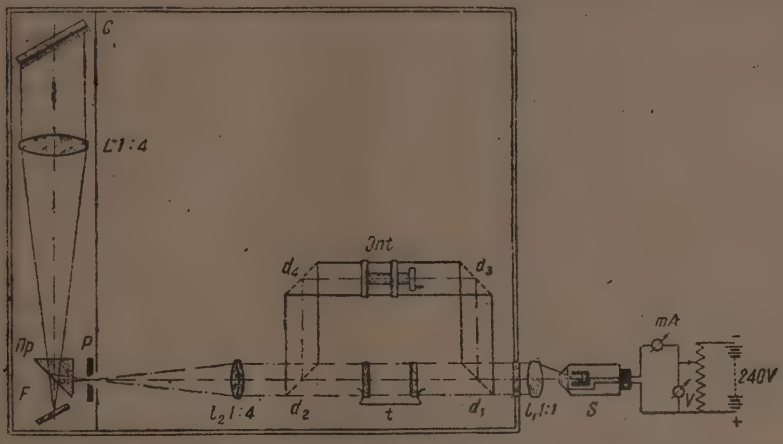

Fig. 1. The set-up scheme for studying the spectral lines

tube was from a direct current $i=20$ mA, $V=185$ volt and its consumption was about 3 watt. When filled with a spectrally pure gas (He, Ne; A; Kr) or with Hg, the source gives an infra-red spectrum of such a brightness that the near infra-red region, up to 8000 Å; can be observed visually (data on the spectral limit of visibility of our lamps are to be published in a paper by N. I. Pinegin). The light issuing from the hollow cathode (end-on) passes through an opening ($d=2$ mm) bored in the anode and is collected by a lens of short focal length l_1. A parallel beam of light traverses the etalon Fabry-Perot or is deflected by a system of mirrors a_1, a_3, a_4 and a_2 to pass through a Hilger's interferometer (large model).

After leaving the interferometer, the beam of parallel rays is collected on to the slit of the spectrograph P by means of the achromatic lens l_2 which thus gives the image of interference fringes localized at infinity.

A small deflecting prism $Пр$ placed in a pencil of diverging rays turns it upon a large lens L. This lens has aspherical surfaces and an aperture of $d:f=100^{mm}:400^{mm}$.

It images the spectrum given by the diffraction grating G (14438 lines per inch) in the focal plane F. The lens was corrected in the far red region to give a satisfactory image of the near infra-red lines. The spectral lines crossed by the interference fringes have been photographed on plates sensibilized with xenocyanine (the xenocyanine was prepared by I. I. Levkoev, of the Institute for Kinematographic and Photographic Research). The linear dispersion of the autocollimation spectrograph is 41.5 Å/mm in the first order.

Owing to the high numerical aperture of this set-up, the focusing and photographing of interference spectra in the infra-red region is a rather simple matter. The many hours of exposure are reduced to but a few minutes. The apparatus is placed in a thermostat upon a ferro-concrete foundation with rubber padding to keep out mechanical and temperature influences.

The elements tried were Hg, (Xe), Cd, Kr, Zn, A, Ne and He. Some even isotopes of these elements must have the spin equal to zero ([3]). Particular attention was given to krypton, as there was a direct invitation from the VII Interna-

tional Conference on Weights and Measures (Paris; 1927) ([4]) to investigate the lines of krypton as the best standard of wave-length. Photographs of the spectrum of Kr were taken up to 10874.92 Å. Part of the spectrum in the region 7000—10 000 Å is shown in Fig. 2 (see the inset).

The structure of spectral lines was studied on etalon with gaps of 10; 20; 40; 62.5 and 125 mm. The mirror planes made as accurately as up to 0.02 of a wave-length were coated by evaporation of silver *in vacuo*. The best of the layers exhibited the following relation between transmission, reflection and absorption: 3: 94.5 : 2.5 per cent for λ=5780 Å.

The mirrors, 5×5 cm in area, mounted in Π-shaped brass etalons were adjustable by screws which exerted pressure upon the mirrors by means of springs.

Of the spectral lines investigated the infra-red line of krypton λ=9751.759 Å ($^1s_4 - {}^2p_{10}$) continued to show the sharpest interference pattern up to the 62.5 mm etalon.

It may be seen from Fig. 3 (see the inset) that with the comparatively large path difference of 125 mm the interference is quite distinct. Its order then is barely $p = \frac{12.5 \times 10^8}{9751} \approx 130\,000$; which is very far from the limiting order of $N = 600\,000$ established by Fabry and Buisson ([5]) on experimenting with visible lines of krypton ($\lambda = 5570$ Å) at room temperature. So there are good reasons to hope that with our infra-red line the interference can be obtained up to a large path difference.

In conclusion the close participation of G. V. Pokrovsky in this work is gratefully acknowledged. Thanks are also due to N. M. Gudris for constant advice.

State Optical Institute.

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CHEMISTRY

THIOSULPHATE COMPOUNDS OF PALLADIUM

By D. I. RIABCHIKOV and A. P. ISSAKOVA

(Communicated by A. E. Arbusov, Member of the Academy, 2. V. 1943)

On the interaction with platinum salts the thiosulphate; as has been shown earlier (1), gives compounds in combination with a different number (1—4) of $S_2O_3^=$ groups.

Further investigation has shown that of the metals of the platinum group no one but palladium; which is the nearest analogue of platinum; forms compounds with thiosulphate.

Just how far goes the analogy between platinum and palladium in their combinations with thiosulphate is the interesting question to be taken up now.

The chloropalladite of potassium on reacting with the thiosulphate in equimolecular ratios yields no complex compound. At room temperature the saturated solutions poured together begin at once to change their initially brown colour to darker tints; and after 2 or 3 minutes the solution is perfectly black because of the precipitation of a powder which turns by degrees into flakes. The black flaky precipitate forms immediately after blending the solutions, if the latter have been preliminarily heated. It can be clearly separated out by filtration. Then the mother liquor is completely colourless, and no palladium is detected in it by the reaction with dimethylglyoxim.

Numerous experiments have revealed a quantitative separation of palladium from the solution. The black precipitate consists of a mixture of PdS_2O_3 with PdS. On boiling the solution with the precipitate the content of palladium sulphide is increased.

A different result is obtained if the amount of thiosulphate is increased to a proportion of two or more molecules for every molecule of potassium chloropalladite.

As we then blend the solutions of the reacting substances; a powder of a yellow colour is seen directly to be carried down.

The reaction runs according to the equation:

$$K_2[PdCl_4] + 2Na_2S_2O_3 \rightarrow K_2[Pd(S_2O_3)_2] + 4NaCl.$$

This precipitate is poorly soluble in water, but is dissolved on heating in an excess of thiosulphate, and the solution that forms is a cherry colour. According to the analysis data, the yellow powder corresponds in composition to the compound $K_2[Pd(S_2O_3)_2]$.

The pink salt of the composition $[PdCl_4][Pd4NH_3]$ is sparingly soluble in water, but dissolves rapidly in the thiosulphate solution at ordinary temperature, whereby free ammonia is separated. The separation of ammonia indicates that the thiosul-

phate residues are introduced not only in the anion of this salt, but also in the cation.

The reaction was carried out with a calculated quantity of the thiosulphate, there being four molecules of the latter to every molecule of the complex palladium salt: $[PdCl_4][Pd4NH_3]:4Na_2S_2O_3$.

With this proportion the pink salt was very rapidly dissolved in the thiosulphate giving rise to an orange-coloured solution. The smell of ammonia about it was insignificant. This means that the reaction whereby the intra-complex groups are displaced by the thiosulphate residues takes place mainly within the anion, according to the scheme

$$[PdCl_4]^{=} + 4S_2O_3^{=} \rightarrow [Pd(S_2O_3)_4]^{6-} + 4Cl^{-},$$

while the cation of this complex salt $[Pd4NH_3]^{++}$ undergoes no change in the solution. Alcohol, however, carries down an abundant precipitate of a bright-yellow colour.

In composition this precipitate corresponds to the compound

$$[Pd(S_2O_3)_4][Pd\,4NH_3]_3.$$

When a solution of the salt $[Pd2NH_3Cl_2]$-trans is acted upon by a calculated quantity of the thiosulphate, the formation of crystalline precipitate sparingly soluble in water and slightly yellow in colour takes place. This product is a compound of the following composition:

$$[Pd\,2NH_3 \cdot S_2O_3 \cdot H_2O].$$

The reaction may be written thus

$$\begin{bmatrix} NH_3 & & Cl \\ & Pd & \\ Cl & & NH_3 \end{bmatrix} + Na_2S_2O_3 + H_2O \rightarrow \begin{bmatrix} NH_3 & & S_2O_3 \\ & Pd & \\ H_2O & & NH_3 \end{bmatrix} + 2NaCl.$$

The thiosulphate residue displaces one of the chlorine atoms, because chlorine is less firmly bound with the central atom of the complex than is the molecule of ammonia. The second atom of chlorine once it has come in trans-position to the thiosulphate residue, proves to be greatly weakened and is displaced by a molecule of water.

If, however, a solution of the salt $[Pd4NH_3]Cl_2$ is acted upon by the thiosulphate on the strict proportion of 1 to 1, a crystalline precipitate of bright-yellow colour appears very soon after the solutions have been blended, and a strong smell of ammonia is felt.

Under these conditions the reaction proceeds according to one of the following three equations:

$$[Pd\,4NH_3]Cl_2 + Na_2S_2O_3 \rightarrow \begin{cases} [Pd\,4NH_3]S_2O_3 + 2NaCl & (1) \\ [Pd\,3NH_3 \cdot S_2O_3] + 2NaCl + NH_3 & (2) \\ [Pd\,2NH_3 \cdot S_2O_3 \cdot H_2O] + 2NaCl + 2NH_3. & (3) \end{cases}$$

The investigation of this process has shown that in the first stage the reaction mainly follows eq. (1) and forms a product of the composition $[Pd4NH_3]S_2O_3$. This compound, owing to the high activity of the thiosulphate residue, is converted into palladium triamminothiosulphate $[Pd3NH_3.S_2O_3]$. Partly, however, the process is directed toward the substitution of a molecule of water for the ammonium molecule situated in trans-position to the thiosulphate group.

In case this reaction is conducted with a twofold amount of thiosulphate, there are two thiosulphate residues getting into the inner sphere, according to the

following equation:

$$[Pd\,4NH_3]Cl_2 + 2Na_2S_2O_3 \rightarrow Na_2[Pd\,2NH_3(S_2O_3)_2] + 2NaCl + 2NH_3.$$

The thiosulphate residues assume then a trans-position. With a solution of the $[Pd4NH_3)]Cl_2$ salt a sparingly soluble crystalline product is formed, $[Pd2NH_3\,(S_2O_3)_2][Pd4NH_3)$, which is a dimer of the formerly obtained compound $[Pd3NH_3 \cdot S_2O_3]$. This reaction offers a complete analogy to what has been observed with platinum salts.

Somewhat different is the character of the reaction if we start from the thioureal derivative of the composition $[Pd4SCN_2H_4]Cl_2$. Upon blending the concentrated solutions of these salts on the basis of their molecular ratio, there at once appears a nice crystalline precipitate of an orange-red colour.

In water the precipitate is dissolved with difficulty and stands well heating up to 110° C. By analogy with the ammoniac form, it was supposed to correspond to the formula $[Pd3SCN_2H_4 \cdot S_2O_3]$. After a more thorough investigation, however, this formula had to give place to the following: $[Pd2SCN_2H_4 \cdot S_2O_3 \cdot H_2O]$.

The formation of this product followed the equation:

$$[Pd\,4SCN_2H_4]Cl_2 + Na_2S_2O_3 + H_2O \rightarrow [Pd\,2SCN_2H_4 \cdot S_2O_3 \cdot H_2O] + 2SCN_2H_4 + 2NaCl.$$

In the compound obtained the thiourea molecules are in trans-position.

With a double amount of thiosulphate the reaction leads to the formation of a soluble product according to the equation:

$$[Pd\,4SCN_2H_4]Cl_2 + 2Na_2S_2O_3 \rightarrow Na_2\left[\begin{matrix} SCN_2H_4 & & S_2O_3 \\ & Pd & \\ S_2O_3 & & SCN_2H_4 \end{matrix}\right] + 3NaCl.$$

By ethyl alcohol an oily liquid is separated from the solution, which crystallizes on careful evaporation over sulphuric acid in a carbon dioxide atmosphere. Its composition corresponds to the formula:

$$Na_2[Pd\,2SCN_2H_4(S_2O_2)_2]\,6H_2O.$$

A peculiar course has the reaction of the thiosulphate with the ethylendiamine compound of palladium having the composition $[Pd2N_2C_2H_8]Cl_2$. This compound was obtained by adding to potassium palladite a somewhat excessive quantity of ethylendiamine, as compared to the theoretical computation, and heating the reacting mixture on a water bath.

The product formed was not separated in the solid state, but to the colourless solution the thiosulphate was added at a rate of two molecules for every atom of palladium.

Upon long standing at room temperature (for 72 hours) the solution took on a greenish-yellow colour. At the bottom of the flask were lying polygonal crystals yellow-greenish in colour, poorly soluble in water. Dried over sulphuric acid to constant weight, this product was found to correspond to the composition $[PdN_2C_2H_8 \cdot S_2O_3]2H_2O$.

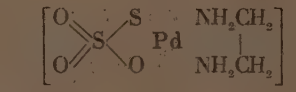
The mother liquor was mixed with ethyl alcohol. Some time later the precipitation of thin, long fibres; yellow in colour, was observed to take place in the alcohol solution. The composition of this precipitate corresponded to the formula $K_2[PdN_2C_2H_8(S_2O_3)_2]$.

Hence the reaction between the thiosulphate and palladium ethylendiamine

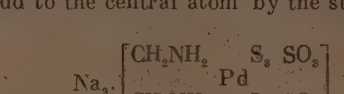
taken in such quantitative relations as were usual in the experiment appears to proceed in two directions:

$$[Pd\,2N_2C_2H_8]\,Cl_2 + 2Na_2S_2O_3 \begin{cases} \nearrow [PdN_2C_2H_8 \cdot S_2O_3] \\ \searrow Na_2\,[PdN_2C_2H_8\,(S_2O_3)_2] \end{cases} + N_2C_2H_8 + 2NaCl.$$

In both cases one molecule of the ethylendiamine is displaced. But in the first case it yields its place to one $S_2O_2^{=}$ group which is added to the central atom by a closed cycle



In the second case the two coordination places are taken up by the individual $S_2O_3^{=}$ groups which add to the central atom by the sulphur atoms only:



It appears from the point of view of structure that as the remaining molecule of ethylendiamine occupies two extreme lateral places, the thiosulphate residues within the sphere can be only in cis-position.

Addition to $[Pd2N_2C_2H_8]Cl_2$ of a large amount of thiosulphate leads to a complete displacement of the ethylendiamine molecules from the inner sphere; they are substituted with the thiosulphate residues.

Conclusions. 1) Palladium salts when acted upon by thiosulphate exhibit a mechanism of interaction essentially similar to that observed with platinum salts.

2) A series of compounds of palladium with thiosulphate have been isolated.

3) Lines have been indicated for their more detailed investigation.

Institute of General and Inorganic Chemistry.
Academy of Sciences of the USSR.

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CHEMISTRY

NEW METHOD FOR DETERMINATION OF DRY RESIDUE IN WATERS, BRINES AND OTHER SALT SOLUTIONS

By I. V. TANANAEV and M. I. LEVINA

(Communicated by V. G. Chlopin, Member of the Academy, 6. VII. 1943)

When the dry residue is determined in salt solutions of different kind, the method of direct evaporation followed by drying and weighing of the residue is usually precluded by the presence of calcium and magnium chlorides in such solutions. These chlorides while passing on evaporation and drying through a series of hydrate forms, undergo a partial decomposition and compose an unconstant weight form. Ordinarily, this obstacle is removed by the evaporation of the test solution with sodium carbonate. This method was at the time thoroughly checked by Efremov [1], and because very accurate results are obtained by it, it has found use in laboratory practice.

There also are some drawbacks, however, with the carbonate method. When sodium carbonate is added, the mixture while subjected to drying must long be kept at a low temperature (60—80° C), because otherwise there is a spray of fine drops flying from its surface. The dry residue containing sodium carbonate in excess is very hygroscopic and certain precautions are required, lest the results of weighing should be affected by the absorption of moisture. For the same reason the sodium carbonate must either be dried just before the experiment or held under special conditions. Finally, the sodium carbonate method cannot be used if the easily hydrolyzed salts of heavy metals are present in the solution.

All of this has compelled us to look for some other substances superior to sodium carbonate and not suffering from the disadvantages enumerated above. Such a substance we found in the sodium fluoride. The latter forms practically insoluble fluorides of calcium and magnesium which are also easy to dry. Nor is it hygroscopic, may readily be obtained in a pure state and, once dried, requires no special conditions of storage. Furthermore, the presence in the solution of easily hydrolyzed salts, such as $BeCl_2$, $AlCl_3$, $FeCl_3$ and the like (in the absence of a free acid), does not prevent the determination of the dry residue by means of NaF, because the cations of these salts form stable double salts with the latter.

Experimental. The use of NaF in determining the dry residue was tried on artificial solutions of $CaCl_2$, $MgSO_4$ and $MgCl_2$ in a given concentration (the method employed in the determination of the dry residue is described below).

For the solution of $CaCl_2$, with 0.2738 g of $CaCl_2$ in the test-sample, there were obtained 0.2740, 0.2735 and 0.2758 g.

For the solution of $MgSO_4$ with 0.2436 g of $MgSO_4$ in the test-sample: 0.2426 and 0.2436 g.

For the solution of $MgCl_2$: 0.2192, 0.2196 and 0.2202 g instead of 0.2170 g $MgCl_2$.

The exaggerated results obtained in the last case have found no explanation so far. However, when natural objects were analysed, this phenomenon was not observed.

To complete the experiment a number of natural salt solutions (brines) of different composition were analysed for their dry residue. For greater confidence in the correctness of the data obtained, we made parallel tests for dry residue by the carbonate method. The results are brought together in the annexed table. In order to give a more complete characteristic of the solutions investigated, as regards their composition, their full analysis is referred to in this table.

Name of sample	K	Na	Mg	Ca	SO_4''	Cl'	HCO_3'	Calculated total	Dry residue	
									Na_2CO_3 method	NaF method
No. 19 lake Baskunchak	0.013	3.297	0.020	0.192	0.455	5.146	0.020	9.143	9.14	9.11
No. 10 lake Tobeichik	0.171	5.750	0.893	0.028	1.902	10.265	0.028	19.037	19.14	19.11
No. 1 » »	0.183	6.070	0.947	0.076	1.970	10.947	0.031	20.271	20.43	20.36
No. 1 springs from Baskunchak	0.028	5.640	0.007	0.185	0.480	8.710	0.017	15.067	15.05	14.93
No. 7 lake Nizhne-Mukhanovsk	0.030	1.400	0.360	0.190	0.410	3.270	0.040	5.700	5.70	5.70
Solution when obtaining posthalat	3.790	0.460	4.140	0.020	20.930	0.710	—	30.050	30.15	30.18
No. 8 lake Tobeichik	0.174	5.789	0.899	0.055	1.906	10.370	0.028	19.221	19.35	19.33
No. 12 lake Ebeity	0.024	10.110	0.703	0.004	8.700	11.230	0.030	30.802	30.78	30.75
No. 54 » »	0.027	10.320	0.641	0.004	9.220	11.000	0.040	31.253	31.24	31.19
No. 55 » »	0.020	10.140	0.693	0.004	9.150	10.910	0.030	30.945	30.73	30.94
No. 49 » »	0.020	10.160	0.684	0.004	9.130	10.930	0.028	30.956	30.84	30.79

It may be easily seen from it that the concurrence of the data obtained by the two methods leaves nothing to be desired. It appears therefore expedient to introduce into practice the use of sodium fluoride instead of the carbonate in determining the dry residue; the more so as the new procedure is much more rapid. Certain fluctuations can possibly take place if the test solution contains boric acid in considerable quantities. But this we have not verified by experiment.

The Procedure of Determining the Dry Residue. A weighed sample of NaF (1—1.5 g) is placed in a weighing bottle with a short glass rod and all is weighed after drying at (it is more convenient to use a preparation of NaF dried beforehand at the same temperature). Afterwards a weighed portion of the brine is put in the same bottle and the weighing is repeated. Under careful stirring with the glass rod the water is removed in a drying chamber at 100—110° C and the temperature is increased to 180° C. An hour later the weight is measured and, if necessary, the drying is continued to constant weight.

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GEOCHEMISTRY

ON THE VARIATION OF ISHIMBAY OILS

By G. D. HALPERN

(Communicated by S. S. Nametkin, Member of the Academy, 27. IV. 1943)

When examining the vast analytical material bearing on the composition and properties of individual oils from the wells of the Southern and Buranchinsky massif of the Ishimbay deposits we soon become aware of this deposit being characterized by zonal oil distribution. Indeed, from the centre to the periphery both the specific weights and the viscosities of the oil increase regularly, the specific weight from 0.8654 to 0.8873, and viscosity with 20° C, from 2.00 to 3.88 E. This distribution is roughly conformable with the structure of the deposit ,stretching along the roof of the Artinskian layer. The lightest oils, both in the Southern and Buranchinsky massif, are confined to the core of the deposit. According to the chemical properties of its oil, the Southern massif stands distinctly apart from the adjoining Kuzminovsky, Buranchinsky and Second field, from which it is separated by a belt of heavier and more viscous oils. The relief of the Artinskian layer is supposed to be the result of the reef having grown on a feebly expressed underlying structure.

The clearly manifest zonality of the oils testifies of a poor communication between the separate areas of the deposit. Even when intense drainage of the latter with worked wells is practised, no mixing up of the oil from different zones, that would lead to a levelling of its properties all over the bed, may be observed. Such a levelling is typical of sandy reservoirs as those of Chechna and Azerbaidjan, where the composition of stratified oils is remarkable for its stability. And this in spite of the fact that the structures of the Caucasian deposits which are obviously tectonic ones, are often much more pronounced than those of the Ishimbay deposit. If the zonal distribution of the oil properties were due to a gradual migration of these oils from the underlying beds, such a phenomenon would long since have been reported for the thoroughly explored regions of the Caucasus. We dispose, however, of no such data.

Along with zonal variations the limestone massifs of Ishimbay are characterized by a high variation of oil within a single well at various periods of its life. This type of oil variation with time has not been reported for the Caucasus; for the Sterlitamak region Vellikovsky and Pavlova have reported it as late as in 1932.

When discussing the various features of Ishimbay oils, we have to point out their being closely related both by their fractional and elemental chemical composition. This makes us ascribe the differences in the viscosity and density of oils to the various degree of polymerism of the tars they contain. We are inclined to consider the polymerization of tars as a secondary metamorphic phenomenon due to the redox conditions being not the same at different zones of the deposit. Let us suppose that in the central zones the medium is a clearly reducing one; in the peripheral zones the effect of a void or water-containing rock may well be appreciably to suppress the reducing processes. Here slight oxidation is likely to take place;

insufficiently intense for destroying the oil, but strong enough to form in low concentration substances that initiate the development of chain polymerization.

We thus are led to a conclusion that the zonal variation of the Ishimbay oils is due to: 1) their being fixed to definite districts of the deposit; and 2) to redox conditions changing gradually from the centre of the deposit towards its periphery.

It should by no means be supposed that the composition of the oil and its features are similar throughout the deposit. As a matter of fact, separate areas within a zone adjoining a well may show a considerable variety of oils. We have as yet discussed only the mean composition and the mean features of oil filling a given zone.

In order to conceive the variation of oil with time, one should consider the peculiarities of the structure of the reefogenic bed of the deposit, and make use of individual rather than average data. The seggregated sedimentary beds of the Caucasus are characterized by a great homogeneity, both along the stratification and in a direction normal to it. Their cavities that are filled with oil are largely determined by the size and degree of seggregation of the clastic material. Here the vacuousness of the rock is adequate to the pore space, the latter being in the first approximation similar in all the directions of the deposit.

Quite different is the picture observed for the limestones of Ishimbay. Instead of the sedimentation of allochtonic material, the main «structure-forming» factor that was operative here is the growth of reefogenic organisms. The growth of the reef shows a rather grotesque pattern, with occasional condensations and rarefactions. Unlike the sedimentary bed whose chief and sometime unique structure-forming element is the plane of sedimentation, complicated cavernous structures appear within the reefogenic bed. Individual cavities may communicate with one another in any direction, in the most fancyful manner, or else they may be fully isolated, separated from one another either by a normal biogenic interstice, or by secondary autochtonic clastic material. All this goes along with usual microporosity caused by spaces between the organisms or fragments composing the limestone.

The peculiar structure of the reservoir results in the creation of individual conditions of oil metamorphization in separate chambers, caverns, which also offer highly variable possibilities of migration towards the drift of the well. Owing to this, a phenomenon may frequently be observed at Ishimbay that seems rather paradoxical at the first sight, viz. that at the initial period of its existence a flowing well yields a heavier oil than at a later period. Making its way through a rock with high gas pressure, the well is fed from separate cavities containing most various oils. According as gas pressure is waning, the specific value of the cavern which contains the most mobile oil increases, and the well is observed to produce a lighter oil during a certain period of time.

After one has passed to the compressor, and further on, the pumping operation of the well, the store of mobile oils near the drift becomes exhausted and; along with lower yields, heavier oils are observed. This is caused by the fact that among the caverns feeding the well such become operative again as contain oils of higher viscosity. Such an increase in the weight of oil along with the operation of the well is most typical of the Ishimbay deposit; it is associated with an increased emulsification, leading to a gradually increasing water content in the humid oils of Ishimbay, even if the ratio of oil and water supplied remains stable.

From what has been said above it is clear that there may occur still more complicate variations of oil supplied by an individual well at different periods of its life. It will largely be due to the freedom with which various oils find access to the well from areas yielding them in a different way.

The second type of oil variation characteristic of the Ishimbay, that of variations displayed by a single well at various periods of its life, finds in this way a reasonable explanation.

The subject will be treated elsewhere in more detail and with addition of cartographic material (Bull. Acad. Sci. URSS, sér. techn.).

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GEOCHEMISTRY

ON THE METHODS OF RADIOACTIVITY MEASUREMENTS. A VACUUM METHOD FOR MEASURING THE CONTENT OF RADON IN WATERS AND GASES

By E. S. STCHEPOTJEVA

(Communicated by V. G. Chlopin, Member of the Academy, 18. V. 1943)

The two methods applied until recently, viz. that implying the use of a fontactometer, and the circulation method ([1], [2]), are not free of a number of defects. In 1938 Prof. I. E. Starik suggested for the above purpose the vacuum method which is commonly applied to solutions. This method has many advantages among which the principal are the accuracy and rapidity of measurements. However, when applied in its ordinary form to the practice of Rn measurements in waters; the use of the vacuum method presented difficulties: in most laboratories were lacking ionization chambers of due dimensions (no less than 2.5—3 l), and it was a rather complicated affair to manufacture a sufficient number of them.

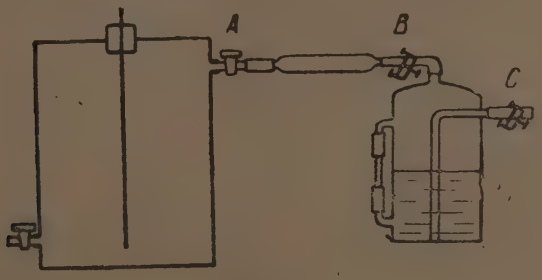

Fig. 1

This difficulty could be surmounted with the help of the following modification of the vacuum method commonly applied. About 100 ml* of water to be tested is sorbed into a preliminarily evacuated small (~200 ml) metallic vessel, the «washer» (Fig. 1)**; the washer is then being shaken for about $^1/_2$ minute, and joined to a previously evacuated ionization chamber of the usual small size. By unfastening the clamps *A* and *B*, air is drawn into the chamber from the washer,

* The precise volume of water is determined after radon has been transferred to the ionization chamber.

** This device has been developed by the operator of the Radiological Laboratory of the Balneological Institute of the Caucasian Mineral Waters, A. Nikolenko, in collaboration with other workers of the same laboratory.

after which clamp *B* is fastened again; while clamp *C* is slightly unfastened in order to admit fresh air (up to atmosphere pressure); after this it is clamped again.

The washer is shaken again; and the air is left out again through clamp *B* into the chamber. The alternating shaking, air transfer from the washer into ionization chamber and inlet of the fresh air into the washer are repeated six times.

If the chamber still contains rarefied air, a continuous jet of air is sent through the washer into the ionization chamber until the latter is filled. The whole procedure of the transfer of radon from the washer into the chamber takes several minutes. Like in the case of the vacuum method, all of the radon contained in the water entirely passes thereby into the ionization chamber. No corrections for the radon left in the water are thus to be introduced.

The vacuum method so modified was widely applied to the practice of measurements of radon content in water by the Mineral Waters Expedition of the Radium Institute in 1940, as well as at the Pjatigorsk Radiological Laboratory. It was thereby found possible to apply it even with chambers from the Universal apparatus([3]), and with chambers from the Schmidt apparatus ([1]). This method is by far more accurate, rapid and practically much more convenient than the circulation method usually applied. Using a single apparatus (one-string electrometer; or Schmidt's electrometer with adjustable chambers); it was found possible to make 20—30 radon measurements per day.

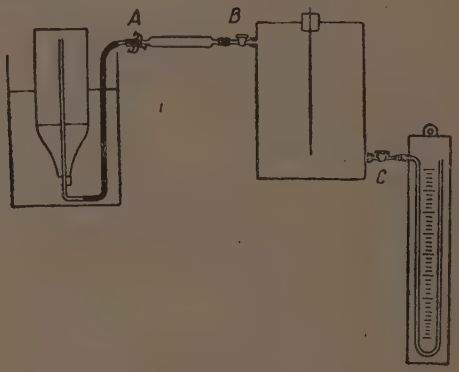

Fig. 2

Two methods of sampling are usually applied to measurements of radioactivity of gases: 1) the gas collected from a source with the aid of a gas-separator, a glass funnel etc., is pumped by means of a rarefying pump through an ionization chamber until the whole chamber is found to be filled with gas. The volume of the gas that is to be let through the chamber should be tenfold that of the chamber. The volume of the gas taken is determined from the volume of the ionization chamber. 2) The gas is collected from the source in a bottle; the latter is overturned in water; and the gas is transferred into an evacuated ionization chamber in the usual way. The volume of the gas introduced into the ionization chamber is determined after the volume of water that has entered the bottle.

The main drawback of the two methods consists in that the volume of the gas to be measured fails to be determined with sufficient accuracy. And this is often caused by an insufficient knowledge of pressure, temperature and humidity conditions that prevail during gas sampling. Besides, if the ionization chamber is entirely filled with the gas to be investigated, an accurate evaluation of the content of radon in the gas becomes a rather difficult matter, since the ionization current created by α-particles in the chamber depends on the composition of the gas filling the chamber.

These defects may be eliminated by using for measurements of radioactivity of gases the following method. The ionization chamber (Fig. 2) is connected on the one side with an open mercury manometer; on the other side it is connected through a tube filled with calcium chloride with a rarefying pump. The chamber is evacuated, and clamp A fastened. After having shut out the pump; the tube containing calcium chloride is connected with a tube whose second end is introduced into a bottle with gas; preliminarily unstopped under water*. The difference between the mercury levels h_1 having been taken on the manometer; clamp A is opened, and the gas from the bottle is let into the chamber so as to fill no more than $^1/_5$—$^1/_4$ of the chamber's volume. Hereupon clamp A is fastened again; and the differences between the levels h_2 taken on the manometer. The readings taken; clamps B and C are adjusted; the tube containing calcium chloride shut out; and fresh air is let into the chamber through a clean tube with calcium chloride until atmospheric pressure is established.

If the volume of the ionization chamber is denoted by V_0; and the temperature of the surrounding air at the point of transfer, by t; it is easy to understand that with 0° C and 760 mm of mercury column the volume of dry gas collected will be

$$V = \frac{V_0 (h_1 - h_2)\, 273}{760\,(273 + t)}.$$

The error in the determination of the gas volume will chiefly depend on the error committed in determining the magnitudes of h_1 and h_2; which cannot exceed 4 mm of the mercury column. The total difference between h_1—h_2 being of the order of 150 mm, the error involved in the determination of the gas volume will thus not exceed 3 per cent.

Radium Institute,
Academy of Sciences of the USSR.

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* If sampling is practised directly from the source, the tube containing calcium chloride may be directly connected with the one leading from the gas-separator or funnel.

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GEOLOGY

RECENT LITHOGENESIS ON CONTINENTAL PLAINS

By B. L. LICHKOV

(Communicated by W. I. Vernadsky, Member of the Academy, 12. IX. 1942)

Within the territories of continental plains the formation of geological deposits proceeds below the soil mantle covering the Earth's surface, being effected by soil aqueous solutions which penetrate through the upper portion of the soil to the so-called «dead horizon». The sedimentary process is thus realized here by illuvial waters, and the «dead horizon» of the soil, as understood by G. N. Vysotsky, is the «live horizon» of the recent lithogenesis. Hence a very important conclusion; viz. that in the interaction between the soil and the rock; it is not the rocks that produce the soil; but on the contrary, the soil cover in the illuvial process that forms new rocks. In this connexion; a new estimation of the age of soils and rocks should be made: the soil cover cannot be considered as derived from the underlying rocks, i. e. as the latest stage of geological evolution. It should be assumed instead that although overlying these rocks, they are actually older than the latter, and have produced them. From this standpoint chernozem, e. g.; is not a derivative of loess hundreds or thousands years old, but in the process of its development has produced the underlying loess and has been in existence since the beginning of the glacial period, i. e. for tens of thousands years. In general, the whole complex of surface continental rocks on the Earth is a derivative of the soil cover and has been produced by the latter; hence the conclusion should be made that the soils have continuously existed during the whole time of existence of the underlying rocks and have even preceded the latter.

If this view be applied to the zonal soils of the European part of the USSR, we see the following.

The enigmatic covering clays of the north which lie on the moraine and can be genetically related to it neither as its deluvium; nor as proluvium for lack of boulders on them; may easily be interpreted genetically with the aid of the illuvial process, and so may the turf iron ores of the north. They have been produced by a thousand-year long accumulation of «gley», i. e. of clays under conditions of a podzol-forming process; the turf ores are due to an equally long accumulation of «orthstein» and «orthsands» in the same process. In the northern and temperate forest zones the process of sedimentation immediately followed the recedence of glacier; thus, the soils here are post-glacial in age, as well as the mantle clays lying on the moraine are produced by forest soils.

South of the zone of mantle clays lying on the moraine; in the steppe zone there occur loesses whose country rock, according to the recent theories, could have been formed either by aqueous, or by aeolian activity, at the expense of the material of the moraine and fluvio-glacial formations. The loess is contemporaneous with the moraine, being a periglacial formation. As found by analysis, the ratio of

carbonates to silicates is equal to unity for loess (B. Polynov); in other rocks that might have served for loess formation, this ratio is but very rarely encountered. In the loesses this ratio may be referred, as suggested by L. Berg (and also by L. Prasolov and S. Neustruev), to weathering under conditions of a steppe climate. In interpreting this phenomenon on the basis of earlier data by N. Bogoslovsky and P. Kossovich; we are to conclude that the loess formed from its country rock by soil solutions which had penetrated illuvially the subjacent country rock from chernozem (Berg's «steppe» weathering). In this sense loess is a derivative of chernozem which from this particular standpoint is contemporary to the glacial period; it is by no means a late post-glacial formation, as it was believed until now. This is evidenced by the presence of a steppe vegetation on the periphery of the glacier, which seems to have grown not directly on loess, but to have always had some soil under it. Throughout the glacial epoch a gradual alluvion of the country rock of loess seems to have proceeded; during the same time this rock was being gradually converted to loess; hence it is clear that chernozem cannot be younger than the loess, but at least as old as the loess and the moraine. Which has been said of the loess of the steppe zone may be said of the loess-like and other loamy rocks of the semi-arid and arid zones enriched with salts: loesses with salt and gypsum; clays with gypsum intercalations, clays with bands of sodium chloride, and of other salts characteristic of the south. All of them are derivatives of an alluvial process; being produced by soils, since the salts have been supplied to them by ground waters from semi-arid and arid grounds. Here, too; as in more northerly zones, the soils are the producers of rocks.

The same holds true of the equatorial zone of tropical forests where; according to the data available, under soils rather poor in humic matter (K. Gedroitz, S. Neustruev) deposits or accumulations of aluminium and iron hydroxides are formed in an illuvial way in the form of laterite beds. The process of their formation may be compared to that of the accumulation during the Tertiary of secondary kaolin. Kaolin is also a result of a soil process under marshy conditions, which is evidenced by the relation of this rock to brown coal deposits.

All of the above data on lithogenesis in different soil zones from the equator to the northern latitudes lead us to conclude that throughout the Quaternary the soils were the producers of rocks on continents; at places covered with vegetation and a soil mantle, where no mechanical sediments, such as deluvial, proluvial and alluvial deposits, could therefore be formed, the surface continental rocks were formed by illuvial processes.

The process of rock formation by the soil mantle on continents is not, however, peculiar to the Quaternary, but went on over the continents during all the preceding history of the Earth. There is therefore good reason to believe that during all periods from the recent epoch to the Cambrian, the surface rocks on continental areas were always derivatives of the soil mantle, and as the latter is a derivative of the climate, to which it is related through the living matter of vegetation, there is reason to believe that the rocks of continents produced by soils from the Cambrian until to-day, are, as shown by W. I. Vernadsky (1940), products of ancient biospheres, being e n t i r e l y due to a peculiar mode of the s e d i m e n t a t i o n p r o c e s s. It should be emphasized here that since for each geological period the soil mantle was zonal, the rocks produced by the soil mantle naturally had to be and were zonal, too. In this connexion it is natural to adopt a zonal arrangement on the continental plains of surface rocks formed by soil continental lithogenesis. The zonal arrangement of these rocks reflects the climatic zoning modified in accordance with that of the vegetable and soil cover, and directly reflects thezoning of aqueous soil solutions. As to the latter, ground waters being inseparable from surface waters, there is good reason to believe that the zoning of the rocks formed by soils is a reflection of that of the surface waters, as has been suggested for lakes by V. Alabyshev and recently emphasized by N. Ivanov («own» waters in the sense of V. Glushkov).

The zonal forms of manifestations and consequences of the illuvial process thus are, in the first place, brought about by the zoning of the surface continental

waters of the Earth. It is clear that this relation has existed on continents at all geological periods.

The illuvial process by which new rocks under soils are formed may take place not only under soils, but under other surface formations as well, such as accumulations of vegetable organic matter, at whose expense deposits of caustobiolites and peat coals, etc. are at various periods formed on our planet. Typical in this respect is the process of kaolin formation under brown-coal bogs of the Tertiary. Analogous to it are the accumulations in recent bogs of clay deposits, formed illuvially; as well as accumulations of bauxites and bauxite-like deposits under coal deposits.

It should be noted that the illuvial process in accumulations of this type of organic remains went on more intensely than under true soils; thicker rock deposits must therefore have been formed in this way under them in the process of lithogenesis. It is known that areas of coal formation were zonally arranged in three belts, being concentrated in forest zones. Two forest belts in temperate zones went parallel to the equator, and one belt along the equator (this has recently been well shown by A. N. Krishtofovich), which corresponds to the present arrangement of forests. It is natural that the underlying coal derivates also show a zonal arrangement. Hence it follows that such coal underlying rocks as «clays», kaolins and bauxites, as well as hypergene iron ores of the bauxite, bauxite-chamosite and other types, must also show zonal arrangement. The zonal arrangement of the latter has recently been pointed out by N. Strakhov. All these formations are a product of the illuvial process. Therefore although underlying coals, they are equivalent to the latter in age, which fact is erroneously not only over-looked, but even disputed.

Hence conclusions of general significance are to be made; viz.:

1) In so far as the coals of the Carboniferous, Tertiary and of all phases in the history of the Earth in general, when coal formation occurred, show zonal arrangement, we are justified in attributing to the zonal arrangement of rocks of the Earth's surface a wider significance than that which could have been assumed from its relation only to the soil mantle.

2) It is evident that the illuvial-infiltrating process on continental surfaces of our planet is one of the normal ways of sedimentary rock formation. In this light we should revise and discard the idea which has become firmly rooted in the science of ore deposits, viz.; that infiltration ores are weathering ores, as opposed to ores of a sedimentary origin. Infiltration ores, i. e. those formed in an illuvial way, should be referred to sedimentary formations, as correctly emphasized by A. Belousov; they should by no means be considered as metasomatic ores.

From what has been said here, great importance of the soil cover and of peat-coaly masses of a similar origin in the formation of new rocks becomes quite evident, in particular the great geological importance of soils as creators of new rocks.

As to the reverse process, the influence of the underlying rock upon the overlying soil, a process still considered as being universal; it should be ascribed a less general significance, being restricted only to the cases of an upward migration of ground waters under the influence of sunrays, which occurs under various climatic conditions, those of tundra included (B. Gorodkov). The influence produced by rocks on soils (not from above, but their lateral effect) is of wide and general significance only under conditions of an intense denudation process: in piedmont areas and areas of near-continental glaciation, where the elements of rocks are supplied in a large quantity to the soil cover, by which they are assimilated.

The influence of soils upon rocks and the formation of rocks by soils in the general course of «soils—rocks» interaction, a process until now overlooked both in soil science and in geology, thus plays a more important and general rôle than the influence of rocks upon soils. This process of rock-formation by soils, as well as of the modifying powerful influence of soils upon rocks, is one of the most outstanding manifestations of the continental sedimentary process on the Earth. We are justified in saying that the illuvial way of sediment formation is as lawful and natural, as the mechanical continental sedimentation in the form of deluvium, proluvium and alluvium.

We are, therefore, induced to revise the question as to the rôle of weathering not only with regard to ore deposits, but of the whole of the Earth's crust. This is due to the fact that in the illuvial process the sedimentation is so closely related to weathering, that it is not possible to draw a sharp boundary between the two. In the general course of the process and in its results they are inseparable. Under these conditions there is no sense in separating from the sedimentary geosphere its upper portion, the weathering crust, as a special geosphere. There are no grounds, whatever, to speak about a separate weathering geosphere which may be distinguished as the upper separate part of the sedimentary geosphere (W. I. Vernadsky, A. E. Fersman, and others). As a matter of fact, on the continents there exists, as shown above, a single sedimentary geosphere related to the biospheres of the past, and within it an upper belt may be distinguished as the belt of sedimentation and weathering of the sedimentary geosphere. Naturally enough, it is sedimentation that prevails here, since it comprises the whole infiltration-illuvial process which has previously been referred to weathering. As to the latter, it should be definitely said, in conclusion, that its significance is not restricted to continental sedimentation: it is of great importance in oceanic sedimentation, where the rôle of soils and peat-vegetable masses is played by bottom silts which have been repeatedly pointed out by W. I. Vernadsky as formations analogous to soils.

Geological Institute of the Tadjik Branch
of the Academy of Sciences of the USSR.

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MINERALOGY

SERPENTINE PSEUDOMORPHS AFTER ACTINOLITE

By V. N. STCHERBINA

(Communicated by V. G. Chlopin, Member of the Academy, 13. V. 1943)

In 1941, during the study of some serpentinite massifs in the Aktiubinsk region, in the Ebeita massif (25—30 km SSW of the town of Orsk), an original mineral formation was encountered that proved to be a large pseudomorph of serpentine after radial actinolite, with a length of the crystals up to 8—10 cm.

Being of a small size (900×300 m), the Ebeita massif is composed of highly uniform antigorite serpentinites. In the southern part of the massif in a nearly latitudinal direction passes a tectonic fracture consisting of two parallel crush zones 15—20 m apart. The central part of both zones is filled with actinolite asbestos forming single veins from 0.25—0.30 to 1.30—1.80 m thick, while the adjacent mass of crushed serpentinites is pierced through by a series of intricately branching veinlets of actinolite asbestos from 2—3 to 10—15 cm thick.

The actinolite asbestos is mainly of the slip-fibre type, but within it there are occasionally observed patches stretched along the length of the veins, which are composed of felted actinolite asbestos. This is an evidence of the tectonic shifts that followed the formation of actinolite asbestos.

A microscopic examination of the crushed serpentinites has shown them to consist of microantigorite which is dissected by strings of cross-fibre (or slightly obliquely fibrous) chrysotile; the extinction of the chrysotile is feebly wavy and in places ripply wavy, which is an evidence of deformation. Within some isolated patches in the thin-sections there occur radially-fibrous or sheaf-like accumulations of large antigorite crystals. Both the microantigorite and the antigorite are cut by minute veinlets of cross-fibre actinolite ($N=1.6285 \pm 0.0015$) which appears to replace the chrysotile, retaining the structure of the latter.

In addition to actinolite veinlets, the thin-sections show some isolated crystals of feathery actinolite, with a prismatic habit, in some places evidently replacing the antigorite. In three of the thin-sections strings of chrysotile possessing no wavy extinction were observed to cut actinolite veinlets.

In the footwall of one of the crush zones dipping SW 210° (80°), between the crushed serpentinites and the vein of actinolite asbestos, there was found a compact, viscous, white or slightly greenish-greyish rock, occasionally massive, occasionally fine, plicated, containing inclusions of a green colour of a prismatic shape and up to 0.5×0.5×1.0 cm in size. Under the microscope the groundmass of the rock is seen to consist of a finely fibrous felt-like matted mass with grey interference colours. In the thin-sections of the massive variety the fibres of the mass are oriented at random, while in the plicated variety a certain parallel arrangement of the fibres is noted. The extinction of the fibres is straight; elongation, positive; refractive index within the range of 1.560—1.565. From optic data, the groundmass of the rock should be referred to serpentine, and to chrysotile rather than to antigorite.

Unusual is its felt-like matted microstructure, possibly disguising the microfibrous structure of chrysotile. In some isolated patches in thin-sections a considerable amount of detached acicular actinolite segregations is observed ($N_1=1.625$, $N_2=1.611$). They occur within a felt-like matted mass of serpentine being arran-

ged at random; and often cut serpentine fibres; which points to a later formation of the actinolite. Under the microscope in transmitted light the above-mentioned grey-coloured inclusions are found to be crystals of a prismatic habit with feather-like terminations along the length of the prism. Under crossed nicols these crystals are conspicuous against the grey groundmass of felt-like matted serpentine, owing to their greyish-blue interference colours and the absence of structure.

Sometimes the impression of an indistinctly pronounced longitudinally fibrous structure is produced by fine needles (feathers) on the ends and sides of the crystals, but under high magnifications it is seen that even in the finest feathers structure is lacking. From optic data ($N_1=1.5715\pm0.0015$; $N_2=1.5695\pm0.0025$; elongation, positive; extinction, straight), the mineral composing the crystals belongs to serpentine (antigorite), but from its feathery shape it resembles actinolite.

Finally, in one of the trenches in the zone of crushed serpentinites there was found a lens composed of a mineral light-green in colour, with a prismatically radial habit, splintery fracture, and silky lustre. The crystals were up to 8—10 cm long; when grown together with their bases they were arranged in four rows (35—40 cm), and grown together with their side surfaces, stretched over 2.70 m. In the central part of the lens a segregation of massive chromite was included, $21\times9\times10$ cm in size (Figs. 1 and 2)*. At some places the finely radial habit of the crystals was replaced by a rough vein-like one.

The chemical analysis of the mineral gave the following data in per cent: SiO_2, 41.09; Al_2O_3, 2.21; Fe_2O_3, 1.80; FeO, 1.85; CaO, 15.38; MgO, 26.35; loss on ignition, 11.57; i. e. results, characteristic of serpentine.

In thin-sections of the finely radial variety it is seen that the mineral is composed of thin longitudinally oriented serpentine fibres, structureless from the inside, with greyish-blue interference colours and a refractive index of $1.5715\pm\pm0.0015$. Occasionally amidst the finely fibrous mass of serpentine there were found elongated crystals similar to those described above in their feather-like shape.

In thin-sections of the coarse-veined variety it was seen that in addition to the finely fibrous serpentine similar to that of the finely fibrous variety, feather-like serpentine crystals are present in a large quantity, as well as feather-like and acicular actinolite segregations ($N_1=1.630$; $N_2=1.613$). In some thin-sections an incomplete replacement of feather-like actinolite crystals by structureless serpentine was distinctly seen. Thus, this lens is a large pseudomorph of serpentine after coarsely crystalline radial actinolite, with a nucleus of massive chromite at the centre.

From an analysis of the above data, the following conclusions may be made.

The processes of serpentinization of the Ebeita massif of ultra-basic rocks near the zone of tectonic fracture took place twice. The first serpentinization caused the formation of the antigorite serpentinites of the whole massif.

The tectonic shifts which occurred thereupon caused the actinolitization of the crush zones, during which there took place the formation of both the actinolite asbestos and the lens of radial actinolite, in the shape of a fringe around the chromite segregation, which was formed probably in the process of the first serpentinization. The new tectonic shifts that took place thereafter caused the formation of the radially fibrous actinolite asbestos and favoured the repeated serpentinization, as a result of which pseudomorphs of serpentine after actinolite were formed.

The case described is interesting because of the clearly established repeated serpentinization of the ultra-basic rocks, the fact of the second serpentinization being confined to fracture zones, and the formation of serpentine pseudomorphs after actinolite. The latter give not only microscopic pseudomorphs, but have also produced a large mineralogical formation. As far as it is known to the author, the case of formation of such large pseudomorphs of serpentine after radial actinolite is described for the first time.

Kazakh Geological Administration,
Alma-Ata.

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* See inset at the end of this issue. The hand specimen shown in the photograph in Fig. 2 is at the Museum of the Kazakh Branch of the Academy of Sciences.

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EXPERIMENTAL MORPHOLOGY

FURTHER EXPERIMENTS ON SUBSTITUTING LOST TEETH BY MEANS OF HOMOPLASTIC TRANSPLANTATION OF TOOTH ANLAGES

By A. G. LAPCHINSKY and A. A. MALINOVSKY

(Communicated by L. A. Orbeli, Member of the Academy, 30. V. 1943)

In our earlier experiments we have obtained healingh-in and normal development in the case of homoplastic transplantation of a tooth anlage into the femur of a rat, into that of a dog, and when replacing teeth extracted from the jaw of an adult dog by tooth anlages of a puppy (1).

When transplanted into the femur, the anlage went on growing during 3—4 months, and as a result a normally developed, mineralized and well rooted tooth was formed. When grafted into a jaw, however, the teeth, though attaining normal size and shape, were distinguished by a somewhat yellowish and dull surface; their rooting into the jaw was besides no perfect one. The toothing proceeded with an excessive rapidity (18—19 days upon the transplantation). True enough, the defects of the tooth surface disappeared in the subsequent, but no fully normal fixation in the socket was attained. And the one of the dogs described, «Norka», lost the tooth after 4 months, probably when eating.

We referred the phenomena described above to an incomplete conformity between the shape and size of the socket and the tooth anlage transplanted, viz., the socket was to wide and not deep enough.

As a result, the anlage approached the surface of the gum and cut it before it had become completely mineralized. Mineralization was completed after the tooth had started functioning. Secondly, the free space around the tooth was filled with fibrin and later on, with connective tissue. Nutrition was, therefore, worse, and rooting less solid that in the case of a good contact with the bony walls of the socket.

This accounts for the fact that the rooting of teeth proceeds in a different way, according to whether the anlage is transplanted into a femur or a jaw: in the former there is created an artificial socket which corresponds to the anlage both in size and shape. The tooth becomes therefore well fixed and its nutrition is normal. Unlike this, when transplanting into a jaw we made use of a natural socket after tooth extraction; it is natural that neither its size nor shape corresponded to the anlage transplanted. As a result, the developing tooth became less solidly rooted and its mineralization was somewhat retarded.

Subsequent experiments went to confirm our assumption that the development will yield quite normal results in the case of a better conformity between the socket and the anlage. This could be attained either by creating an artificial socket instead of the atrophied one, or by a happy choice of a socket that would fit the shape of the anlage.

Having been successful in creating an artificial socket in the case of transplantation of an anlage into the femur, we now chose the other way, and fully

succeeded in our object. The transplantation described below may serve as an example.

By the method described in our previous note we transplanted into the socket of the anterior root of the «carnivorous» tooth (first molar) of an adult dog an anlage of the first premolar (constant) of a young dog 3 months of age. The socket perfectly suited the anlage which touched the bottom of the alveola and closely adhered to its walls. The mucous was sutured in the usual way.

During four months no changes could be observed. Towards the end of this period a well mineralized tooth started cutting. After the process of cutting had been accomplished, a normally developed and well fixed premolar could be observed. The fact that the shape of the tooth (first premolar) did not correspond to its position (place of the first molar) makes indubitable its development from the anlage. The animal made normal use of the tooth which was not loose; there was neither painfulness nor traces of inflammation at the spot of the graft transplanted. After a fortnight the dog perished (has been strangled by its chain).

Fig. 1 represents the preparation of the lower jaw of the dog with a tooth developed out of an anlage of the first premolar at the place of the first molar. Both size and character of the surface of the tooth are perfectly similar to those of the respective own tooth of the dog.

An X-ray photograph of the jaw (Fig. 2) showed a normal development of the root. Its walls are somewhat thinner as compared with the other teeth, which is accounted for by its being younger; normally the walls continue to grow thicker during several years after their development has been achieved (Figs. 1 and 2, see inset at the end of this issue).

It should thus be considered as experimentally proved on higher mammals (dogs) that removed or lost teeth can be replaced by means of homoplastic transplantation of tooth anlages taken from young individuals. This suggests the idea to extend the experiments to the substitution of deficient teeth in human subjects. If successful, such a substitution may be applied on a wide scale in odontological practice.

The importance of this achievement for the restoring surgery may scarcely be overestimated, especially at the actual time of war. Now we have at our disposal a single case of transplantation in man, which did not led as yet to the effect desired. The transplantation was made to one of the authors at the Moscow Stomatological Institute which charged itself of the clinical side of the experiment. Patient 32 years old, development and data of clinical examination normal. Frequent tonsillitis. Transplantation practised at the spot of the second inferior right molar. Tooth affected by caries, entirely decayed. The rootlets were preserved in order to keep back the atrophy of the alveola. Transplantation made on 22. IV. 1942. Surgeons Prof. A. I. Evdokimov and Dr. O. G. Kusel. Remains of roots removed, interstice bit off, socket deepened with the help of the boring-machine. Into the socket thus obtained has been planted the anlage of the first molar (first set) from a newborn girl which died from asphyxy; Blood group similar (III), weight 3100 g. The anlage was being preserved during 56 hours in the isolated jaw smeared with iodine. Considerable bleeding recorded. The operational wound covered with a piece of mobilized mucous. After a few days no oedema at the place of operation; no pain.

On July 13 a sequestrum appeared at the place of operation, presenting the remains of an enamel fragment of the anlage. Neither suppuration nor any important inflammatory phenomena were observable.

The considerable period during which the transplanted anlage held in place seems to mean that there is prospect of further success. The fact that no healing-in took place should probably be referred to the excessively long preservation of the anlage (56 hours), which was due to technical difficulties. The absence of an intense local reaction also makes us believe that healing-in of the anlage is quite possible. Further experiments must check the validity of these considerations.

Before concluding it should be mentioned that, extending our experiments to new objects, we succeeded in obtaining normal development of a tooth from

a tooth anlage transplanted homoplastically into the femur of a cat in the same way as had already been reported [1] with regard to transplantations of tooth anlages to a dog and a rat. This should be taken to mean that the faculty of normally developing homoplastically transplanted tooth anlages is of rather wide occurrence among mammals; for the time being, it has already been discovered for the rat, dog and cat. This justifies us in expecting positive results in the case of homoplastic transplantations in human subjects.

The experiments described above give us right to draw the following conclusions:

1. Homoplastic transplantations of tooth anlages in place of removed teeth give satisfactory results in dogs in case adequate method is applied.

2. Homoplastic transplantations of tooth anlages into bones give equally successful results in the rat, dog and cat.

We feel therefore confident that analogous experiments will yield positive results in man, as well.

Institute of Cytology, Histology and Embryology.
Academy of Sciences of the USSR.

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Зак. 215.

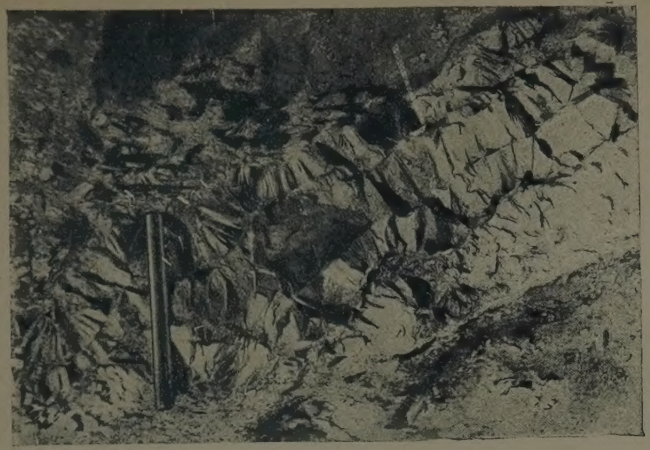

Fig. 1

Fig. 2

Fig. 1

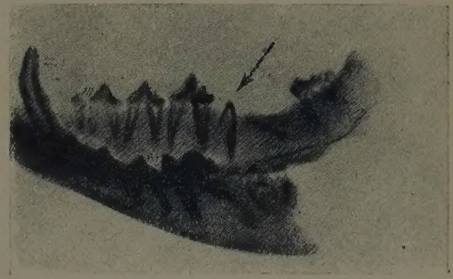

Fig. 2

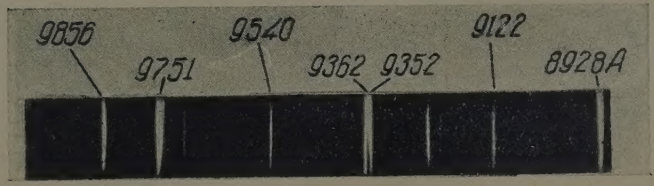

Fig. 2

Fig. 3

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